MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY.

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Professor H. H. TURNER, D.Sc., F.R.S., PRESIDENT, in the Chair.

Major C. F. Close, C.M.G., R.E., Brompton Barracks, Chatham; and
Thomas Andrew Common, 63 Eaton Rise, Ealing, W.

were balloted for and duly elected Fellows of the Society.

The following candidates were proposed for election as Fellows of the Society, the names of the proposers from personal knowledge being appended:

Alexander John Samuel Adams, Superintendent and Technical Officer, Post Office Telegraphs London Central, E.C. (proposed by William Ellis);
Captain Arthur ffolliott Garrett, R.E., Craigbeg, Kingussie, Scotland (proposed by H. H. Turner);
P. Groves-Showell, L.C.C. School for Marine Engineering, Poplar, E. (proposed by Sir W. Christie);
George Bruce Halsted, M.A., Ph.D., Professor of Mathematics and Astronomy, Kenyon College, Gambier, Ohio, U.S.A. (proposed by Professor C. S. Howe);
William T. Litton, Head Master, "Shaftesbury" Training Ship (proposed by Thomas Lewis);
Alfred Noël Neate, Civil Engineer, 34 Prescot Street, New Brighton, Cheshire (proposed by R. C. Johnson);
Alexander Durie Russell, B.Sc., Mathematical Master, High School, Falkirk, Scotland (proposed by P. S. Hardie);

John James Steward, F.R.Met.Soc., Optician, 457 West Strand, W.C. (proposed by William Shackleton) ;
Lewes H. Tamplin, F.R.Met.Soc., Indo-China Steam Navigation Co., Wuhu, China (proposed by C. H. Brewitt Taylor) ; and
David Wylie, Teacher of Mathematics, 9 East Road, Lancaster (proposed by Joshua Jukes).

One hundred and sixty-eight presents were announced as having been received since the last meeting, including, amongst others :

R. Buchanan, The Mathematical Theory of Eclipses (presented by the Author) ; Groningen Astronomical Laboratory, Publications, Nos. 12, 13 (presented by the Laboratory) ; Annals of Harvard Observatory, vol. xlvi. pt. 2, vol. liii. Nos. 1, 3, vol. lvi. No. 1 (presented by the Observatory) ; Catalog der Astronomischen Gesellschaft [Leiden and Wien-Ottakring Zones] (presented by the Society) ; Milan Observatory, Albategnius, Opus Astronomicum, latine versum (presented by the Observatory).

Sixty Charts of the Astrographic Chart, presented by the Royal Observatory, Greenwich, and fifty-nine Charts presented by the French Minister of Public Instruction ; twenty-eight Cartes Autographiées (presented by the Toulouse Observatory) ; seven great enlargements from photographs of the Moon by MM. Loewy and Puiseux (presented by the French Minister of Public Instruction) ; series of twenty-four transparencies from the late Dr. Roberts's Photographs of Nebulæ (presented by Mrs. Isaac Roberts).

Wire Micrometer by Dallmeyer (presented by Mrs. Irving Noble) ; Old Ring Dial (presented by Dr. Little).

Magnetic Disturbances, 1882 to 1903, as recorded at the Royal Observatory, Greenwich, and their Association with Sun-spots. By E. Walter Maunder.

1. *Material Employed.*

Nearly a year ago I prepared a paper on the "Great Magnetic Storms, 1875 to 1903," which was published in the *Monthly Notices*, vol. lxiv. No. 3, p. 205. I confined my examination on that occasion to the instances in which the magnetic movement had amounted to one degree in declination, that being the definition of a "great" storm adopted by Mr. William Ellis, F.R.S., in his paper "On the Relation between Magnetic Disturbance and the Period of Solar Spot Frequency"

(*Monthly Notices*, vol. lx. No. 2, pp. 142–157). But the number of "great" storms recorded in thirty years was only nineteen. I desired, therefore, to extend the examination as soon as possible to a much larger number, and decided to take as the basis of my inquiry all disturbances for which reproductions of the photographic traces were given in the Greenwich volumes. These reproductions have been given regularly since 1882, so that the inquiry commences with that year. The disturbances represented in the plates number about 310 in all, the precise number depending upon the way in which certain long-continued periods of agitation are treated, whether as single disturbances or as series of separate ones, following each other at short intervals.

The first step was to draw up a table of the disturbances to be examined, and I am anxious to state here that the tables upon which all the subsequent work was done were prepared first of all, before the slightest attempt was made to start any inquiry depending upon them. The times of the commencement of the storms, the duration of the storms, the amplitude of the movements of the magnets, and the character of the initial movement all were determined purely from the examination of the plates, before any other investigation was commenced, and in no case have any of these been altered since.

This first table included 310 disturbances; but on its completion it occurred to me that since the selection of disturbances to be represented in the Greenwich plates had not been made with sole reference to the amplitude of the movement it would be worth while to secure an approximation to uniformity by rejecting a few considerably below the rest in range. A range of 20′ in declination was therefore taken as the smallest to be considered, and Table I., containing 276 disturbances, was prepared. The descriptive notes in the Greenwich volumes were also examined for disturbances exceeding 20′ in amount, but not shown in the plates, and a supplementary table was prepared of these, about forty in number. When these are taken into account, although no very broad line of distinction can be drawn between the smallest included and the largest neglected, it is certain that no disturbance of even third-rate importance has been omitted.

A rough classification was next made on the following plan :

Rank of Disturbance.	Greatest Recorded Declination.	Magnetic Movement. Horizontal Force. c.g.s. value $\times 10^5$
Great ...	Greater than 60′	Greater than 300
Very active	Less than 60′	Less than 300
	Greater than 40′	Greater than 200
Active ...	Less than 40′	Less than 200
	Greater than 30′	Greater than 150
Moderate	Less than 30′	Less than 150
	Greater than 20′	Greater than 100

This classification is essentially Mr. Ellis's, except that I have divided his "active" class into "active" and "very active," and that I have only concerned myself with the more considerable of his "moderate" disturbances. The rank of the disturbance is indicated in the second column of Table I. by the letters G, V, A, and M; signifying "great," "very active," "active," and "moderate" respectively. The only other column in the table requiring explanation is the sixth, headed 'Beginning." In cases where the movement began with the sharp characteristic to-and-fro twitch, described in my former paper, the letter S has been placed in this column. In these instances the time of beginning is given to the tenth of an hour.

TABLE I.

Magnetic Disturbances of 20′ and upwards in Declination, as given in the plates in the "Greenwich Observations."

Ref. No.	Class.	Period of Disturbance. Greenwich Civil Time. From		To		Duration in Hours.	Beginning.	Extreme Amplitude of Movements. Dec.	H.F.	V.F.	No. of Rotation.	Heliographic Coordinates of Centre of Sun's Disc. Longitude.	Latitude.
			d h		d h	h		′				°	°
		1882.		1882.									
1	A	Jan.	19 17	Jan.	20 12	19	...	36	·006	·003	378	163·1	−5·1
2	M	Feb.	1 13	Feb.	2 12	23	...	35	·005	·003	379	354·2	−6·2
3	V		20 6		21 1	19	...	42	·009	·002	...	107·8	−7·1
4	G	Apr.	16 23·5	Apr.	17 23	24	S	60+	·030+	·022+	381	93·0	−5·4
5	G		20 3·6		21 8	28	S	70+	·020+	·008	...	51·2	−5·0
6	M	June	15 3·1	June	15 12	9	S	23	·005	·002	383	30·9	+1·3
7	V		24 13		25 7	18	...	50	·017	·007	384	266·3	+2·3
8	A	July	30 23	Aug.	1 11	36	...	33	·009	·004	355	145·4	+5·8
9	A	Aug.	4 15·9		5 11	19	S	31	·018	·006	...	82·1	+6·2
10	A	Sept.	12 3·0	Sept.	12 11	8	S	18	·012	No. reg.	387	293·9	+7·2
11	G	Oct.	2 9·7	Oct.	3 3	17	S	60	·014	No. reg.	...	26·3	+6·6
12	A		5 18		6 16	22	...	32	·012	·003	388	342·1	+6·4
13	V	Nov.	11 21	Nov.	15 16	91	...	50	·012	·004	389	212·4	+3·0
14	M		16 8·3		17 10	26	S	20	·005	·001	...	153·5	+2·5
15	G		17 10·3		21 4	90	S	110	·050+	·025	...	139·2	+2·4
16	V		21 16		22 0	8	...	40	·010	·002	...	83·4	+1·9
17	A		25 16·5		26 3	10	S	37	·008	·003	...	30·4	+1·3
18	A	Dec.	20 15	Dec.	22 1	34	...	34	·007	·003	390	61·7	−1·8
		1883.		1883.									
19	M	Jan.	25 18	Jan.	27 4	34	...	26	·005	·001	392	306·0	−5·7
20	A	Feb.	1 18	Feb.	5 4	82	...	37	·007	·002	...	213·9	−6·2
21	A		22 16		23 8	16	...	40	·007	·002	393	298·5	−7·1
22	V		24 13·7		25 12	22	S	59	·015	·004	...	273·4	−7·2

Ref. No.	Class.	Period of Disturbance. Greenwich Civil Time. From		To		Duration in Hours.	Beginning.	Extreme Amplitude of Movements. Dec.	H.F.	V.F.	No. of Rotation.	Heliographic Coordinates of Centre of Sun's Disc. Longitude.	Latitude.
		d	h	d	h	h		′				°	°
		1883.		1883.									
23	M	Feb. 27	16	Mar. 2	22	78	...	25	·006	·002	...	232·6	−7·2
24	M	Mar. 26	21	28	18	45	...	28	·006	·002	394	234·0	−6·7
25	V	Apr. 3	9·0	Apr. 4	23	38	S	55	·010	·006	...	135·0	−6·3
26	M	19	18	20	9	15	...	25	·003	·002	395	279·0	−5·1
27	A	24	19	26	9	38	...	36	·011	·002	...	212·3	−4·7
28	M	May 20	19	May 22	8	37	...	28	·009	·002	396	228·6	−1·8
29	M	June 30	5	July 2	4	47	...	20	·008	·002	397	53·7	+3·0
30	M	July 8	15·0	11	12	69	S	29	·008	·002	398	302·3	+3·8
31	M	11	17·4	12	8	15	S	28	·008	·002	...	261·4	+4·1
32	M	29	23·8	Aug. 2	7	79	S	24	·007	·003	...	19·9	+5·7
33	M	Aug. 18	16	19	4	12	...	22	·003	·002	399	119·5	+6·9
34	V	Sept. 16	2·7	Sept. 17	16	37	S	50	·018	·004+	400	103·7	+7·1
35	A	18	15·6	19	11	19	S	20	·005	·001	...	70·2	+7·1
36	V	Oct. 5	13	Oct. 6	10	21	...	40	·013	·003	401	207·2	+6·4
37	A	16	16·1	17	11	19	S	32	·006	·003	...	60·5	+5·7
38	M	19	19	20	6	11	...	18	·005	·002	...	19·3	+5·4
39	A	Nov. 1	17	Nov. 3	15	46	...	36	·010	·003	402	208·9	+4·2
40	M	19	21	20	18	21	...	26	·005	·002	403	329·4	+2·1
41	A	22	2	24	0	46	...	32	·009	·003	...	300·3	+1·8
		1884.		1884.									
42	M	Feb. 23	14	Feb. 26	1	59	...	29	·006	·001	406	148·8	−7·1
43	M	29	19	Mar. 4	1	78	...	28	·007	·002	...	67·0	−7·2
44	M	Mar. 28	19	29	12	17	...	22	·006	·001	407	58·0	−6·7
45	M	Apr. 17	15	Apr. 18	12	21	...	22	·005	·002	408	156·2	−5·2
46	M	24	15	25	12	21	...	29	·006	·002	...	63·8	−4·6
47	M	June 22	21	June 24	8	35	...	21	·007	·002	411	359·9	+2·2
48	A	July 2	19·3	July 4	10	39	S	36	·012	·007	...	228·5	+3·3
49	M	Aug. 8	13	Aug. 10	0	35	...	20	·005	·001	412	102·6	+6·4
50	M	Sept. 17	13	Sept. 19	11	46	...	25	·007	·002	414	294·0	+7·1
51	A	Oct. 1	21·9	Oct. 3	9	35	S	30	·012	·003	...	104·4	+6·6
52	V	Nov. 1	12	Nov. 4	8	68	...	41	·012	·004	415	60·9	+4·1
53	M	Dec. 22	18	Dec. 23	12	18	...	21	·006	·001	417	105·6	−2·1
		1885.		1885.									
54	A	Jan. 22	16	Jan. 23	0	8	...	36	·007	·002	418	58·4	−5·5
55	M	Feb. 5	13·5	Feb. 6	8	19	S	18	·006	·001	419	235·4	−6·5
56	V	Mar. 15	8	Mar. 16	6	22	...	52	·008	·009	420	97·9	−7·1

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		d	h	d	h	h		′				°	°
		1885.		1885.									
57	M	May 10	6·2	May 10	20	14	S	24	·009	·002	422	79·7	3·0
58	V	13	13	14	3	14	...	38	·014	·003	...	36·2	−2·6
59	V	25	13·6	28	17	75	S	32	·020	·002	423	237·2	−1·2
60	A	June 24	22·5	June 26	9	34	S	30	·012	·003	424	195·3	+2·4
61	M	Aug. 1	8	Aug. 2	7	23	...	22	·008	·002	425	60·4	+6·0
62	M	Sept. 4	14	Sept. 5	13	23	...	23	·008	·002	427	327·7	+7·2
63	A	15	13	17	1	36	...	30	·006	·001	...	182·9	+7·2
64	M	22	13	24	8	43	...	28	·005	·001	...	90·5	+7·0
65	M	Nov. 10	13	Nov. 12	1	36	...	22	·007	·002	429	164·3	+3·1
66	M	Dec. 7	14	Dec. 8	11	21	...	23	·005	·001	430	167·8	−0·2
		1886.		1886.									
67	A	Mar. 18	19	Mar. 20	12	41	...	30	·010	·001	434	274·8	−7·0
68	G	30	8·2	Apr. 1	3	43	S	65	·020+	·007	...	122·5	−6·5
69	A	Apr. 11	15	15	11	92	...	32	·009	·003	435	320·3	−5·8
70	V	May 8	19	May 9	11	16	...	43	·012	·004	436	321·5	−3·2
71	M	June 22	14	June 23	11	21	...	22	·006	·002	437	88·8	+2·1
72	A	29	18	30	8	14	...	27	·009	·002	438	353·9	+2·9
73	V	July 27	18	July 28	12	18	...	45	·013	·007	439	343·5	+5·6
74	M	Aug. 23	20	Aug. 24	12	16	...	28	·007	·001	440	345·4	+7·0
75	A	Sept. 9	13	Sept. 14	0	107	...	22	·010	·002	...	124·6	+7·2
76	A	Oct. 6	17	Oct. 11	8	111	...	40	·008	·002	441	126 0	+6·3
77	M	Nov. 2	14·7	Nov. 7	0	105	S	26	·007	·002	442	131·2	+4·1
78	A	30	12·5	30	21	9	S	32	·005	·001	443	123·3	+0·7
		1887.		1887.									
79	M	Feb. 12	18	Feb. 15	1	55	...	26	·006	·001	446	225·6	−6·8
80	M	Apr. 4	14	Apr. 9	2	108	...	24	·006	·001	448	275·7	−6·2
81	M	Aug. 1	11·1	Aug. 2	7	20	S	24	·007	·002	452	143·4	+5·9
82	M	28	20	30	11	39	...	22	·005	·001	453	141·7	+7·2
83	V	Sept. 25	16	Sept. 28	8	64	...	47	·009	·005	454	134·2	+6·9
84	M	Oct. 22	14·8	Oct. 23	22	31	S	28	·008	·002	455	138·5	+5·1
85	M	26	10	27	1	15	...	27	·008	·001	...	88·5	+4·8
86	A	Nov. 21	9	Nov. 22	0	15	...	36	·008	·001	456	106·2	+1·9
		1888.		1888.									
87	V	Jan. 13	13	Jan. 15	4	39	...	42	·009	·003	458	125·9	−4·5
88	A	23	16	24	4	12	...	37	·008	·002	459	352·7	−5·5
89	M	Mar. 15	13	Mar. 18	23	82	...	25	·006	·003	460	29·6	−7·1

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		d	h	d	h	h		′				°	°
		1888		1888.									
90	A	Apr. 11	8	Apr. 14	23	87	...	36	·007	·002	461	36·0	−5·7
91	M	May 7	3	May 10	21	90	...	25	·006	·001	462	55·2	−3·3
92	A	20	12	22	0	36	...	30	·009	·004	463	238·3	−1·8
93	M	June 3	14	June 4	11	21	...	29	·006	·002	...	51·9	−0·1
94	M	Aug. 3	19	Aug. 4	7	12	...	20	·006	·001	466	321·9	+6·1
95	M	16	3	16	23	20	...	22	·006	·001	...	159·0	+6·8
96	M	Oct. 19	19	Oct. 22	0	53	...	29	·006	·001	468	24·2	+5·3
97	A	30	19·7	Nov. 1	12	40	S	35	·007	·001	469	239·8	+4·3
98	M	Nov. 16	19	17	23	28	...	24	·006	·001	...	16·0	+2·4
99	M	Dec. 24	4	Dec. 24	22	18	...	22	·004	·001	471	243·6	−2·4
		1889.		1889.									
00	M	Jan. 20	13	Jan. 21	4	15	...	27	·003	·002	472	243·0	−5·3
01	M	Mar. 6	10	Mar. 6	19	9	...	19	·005	·001	473	12·1	−7·3
02	M	17	20	18	1	5	...	30	·006	·002	474	221·7	−7·1
03	M	28	6	29	1	19	...	28	·008	·001	...	84·3	−6·6
04	A	July 17	4·9	July 17	20	15	S	25	·011	·001	478	57·0	+4·8
05	M	Aug. 13	5	Aug. 13	23	18	...	23	·004	·001	479	60·0	+6·7
06	M	Sept. 8	23	Sept. 11	1	50	...	22	·008	·001	480	66·5	+7·3
07	M	22	13	23	3	14	...	20	·005	·002	481	247·2	+7·0
08	M	Oct. 5	17	Oct. 6	5	12	...	29	·004	·001	...	73·4	+6·4
09	M	18	18	18	23	5	...	22	·005	·001	482	261·4	+5·4
10	M	20	16	21	2	10	...	20	·006	·001	...	236·2	+5·3
11	A	Nov. 1	6	Nov. 2	22	40	...	22	·010	·004	...	83·4	+4·1
12	A	26	15	29	0	57	...	34	·006	·001	483	108·8	+1·2
		1890.		1890.									
13	M	Jan. 3	19	Jan. 4	4	9	...	20	·004	·001	485	326·0	−3·5
14	M	Aug. 14	13·9	Aug. 16	11	45	S	18	·006	·001	493	264·2	+6·7
15	M	Sept. 6	12	Sept. 7	10	22	...	18	·003	·001	494	321·4	+7·3
16	M	Oct. 5	13	Oct. 6	5	16	...	24	·005	·001	495	298·1	+6·4
17	M	17	14	19	11	45	...	28	·006	·002	...	139·3	+5·6
18	A	Nov. 7	21	Nov. 8	15	18	...	34	·008	·002	496	218·5	+3·5
19	M	9	13	9	21	8	...	22	·004	·001	...	196·5	+3·3
		1891.		1891.									
20	A	Feb. 11	18	Feb. 15	11	89	...	25	·009	·002	499	35·3	−6·8
21	A	Mar. 2	1·9	Mar. 3	0	22	S	23	·010	·003	500	153·8	−7·3

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		d	h	d	h	h						°	°
		1891.		1891.									
122	M	Mar. 31	14·0	Apr. 1	12	22	S	27	·008	·002	501	124·9	−6·5
123	A	Apr. 8	15	8	23	8	...	35	·007	·002	...	18·8	−6·0
124	M	9	12	9	22	10	...	20	·007	·002	...	7·2	−5·9
125	A	12	14	13	12	22	...	33	·010	·003	502	326·5	−5·7
126	V	May 13	8·3	May 17	3	91	S	50	·020	·007+	503	280·2	−2·7
127	M	June 14	8·7	June 14	18	9	S	23	·008	·002	504	216·3	+1·2
128	M	Aug. 28	22	Aug. 30	6	32	...	24	·005	·003	507	297·0	+7·2
129	A	Sept. 9	10·3	Sept. 12	7	69	S	34	·008	·003	...	145·0	+7·3
130	A	28	12	29	12	24	...	34	·009	·002	508	253·2	+6·8
131	A	Oct. 23	12	Oct. 27	10	94	...	30	·009	·002	509	283·4	+5·1
132	A	Nov. 20	0	Nov. 21	23	47	...	36	·007	·001	510	280·8	+2·0
133	A	Dec. 7	11·2	Dec. 7	22	11	S	24	·010	·004	...	50·6	−0·2
		1892.		1892.									
134	V	Jan. 4	18·8	Jan. 6	10	39	S	40	·009	·003	511	37·4	−3·6
135	G	Feb. 13	5·5	Feb. 14	18	37	S	70+	·029+	·015+	513	238·2	−6·8
136	M	15	13	16	0	11	...	18	·008	·002	...	207·7	−6·9
137	M	20	19·1	21	3	8	S	24	·007	·002	...	138·6	−7·1
138	A	27	0	27	22	22	...	31	·008	·002	...	56·9	−7·2
139	V	29	21	Mar. 5	5	104	...	40	·014	·003	...	18·9	−7·2
140	V	Mar. 6	9·7	9	5	67	S	48	·017+	·007	514	306·0	−7·3
141	G	11	22·5	13	5	30	S	75	·026	·006+	...	233·2	−7·2
142	M	24	20	26	4	32	...	28	·008	·002	...	63·2	−6·8
143	V	Apr. 25	15	Apr. 27	7	40	...	58	·018	·008	515	3·5	−4·5
144	V	May 1	4	May 2	23	43	...	42	·012	·003	516	290·3	−3·9
145	M	16	22·0	17	12	14	S	20	·006	·001	...	82·0	−2·2
146	G	18	8·4	19	8	24	S	73	·016	·011	...	63·3	−2·0
147	M	19	15·4	19	23	8	S	16	·008	·002	...	45·9	−1·9
148	V	June 2	14	June 3	11	21	...	40	·016	·004	517	221·5	−0·2
149	V	27	5·0	28	0	19	S	40	·017	·010	518	255·6	+2·7
150	V	July 12	17·9	July 14	11	41	S	29	·028	·003	...	49·8	+4·3
151	G	16	12·6	17	20	31	S	75	·036+	·007+	519	359·9	+4·7
152	M	Aug. 3	14·0	Aug. 3	23	9	S	19	·007	·001	...	121·0	+6·1
153	M	4	14·5	5	1	11	S	25	·008	·003	...	107·8	+6·2
154	G	12	12·6	13	2	14	S	65	·034	·012	...	2·8	+6·6
155	M	Sept. 5	23·4	Sept. 6	7	8	S	12	·004	·001	520	39·9	+7·3
156	A	Oct. 17	14	Oct. 19	12	46	...	32	·010	·002	522	210·7	+5·5

Ref. No.	Class.	Period of Disturbance. Greenwich Civil Time. From		To		Duration in Hours.	Beginning.	Extreme Amplitude of Movements. Dec.	H.F.	V.F.	No. of Rotation.	Heliographic Coordinates of Centre of Sun's Disc. Longitude.	Latitude.
		d	h	d	h	h		′				°	°
		1892.		1892.									
157	V	Nov. 4	2·5	Nov. 5	12	34	S	46	·011	·005	523	339·9	+3·8
158	V	Dec. 4	20·3	Dec. 5	5	9	S	36	·016	·006+	524	294·6	+0·1
159	M	5	14	6	1	11	...	27	·006	·001	...	284·7	0·0
		1893.		1893.									
160	M	Jan. 5	13	Jan. 6	4	15	...	28	·008	·002	525	236·8	−3·8
161	M	21	15	22	12	21	...	26	·005	·001	...	25·0	−5·4
162	A	Feb. 4	17	Feb. 6	12	43	...	36	·008	·003	526	199·6	−6·4
163	M	Mar. 14	15	Mar. 15	2	11	...	25	·006	·002	527	60·2	−7·1
164	M	15	15	15	23	8	...	22	·004	·001	...	47·0	−7·1
165	M	25	4·5	25	22	17	S	23	·008	·001	528	280·8	−6·8
166	M	26	7·9	27	6	22	S	26	·007	·004	...	265·9	−6·7
167	A	Apr. 26	16·4	Apr. 27	9	17	S	28	·012	·002	529	212·2	−4·4
168	M	June 9	13·0	June 10	7	18	S	22	·008	·002	531	351·8	+0·6
169	A	18	13	20	11	46	...	20	·009	·002	...	232·7	+1·7
170	V	July 15	22	July 16	9	11	...	46	·012	·004	532	230·4	+4·6
171	M	21	14	22	10	20	...	24	·008	·002	...	155·4	+5·1
172	A	Aug. 6	3·9	Aug. 7	23	43	S	28	·015	·005	533	309·3	+6·3
173	A	18	11	18	23	12	...	30	·014	·008+	...	146·9	+6·9
174	M	Sept. 8	0·9	Sept. 10	0	47	S	20	·006	·002	534	235·0	+7·3
175	M	26	12	27	11	23	...	28	·007	·002	535	351·3	+6·8
176	A	29	13·9	Oct. 1	1	35	S	28	·009	·002	...	310·6	+6·7
177	V	Nov. 1	15	Nov. 2	12	21	...	40	·009	·004	536	234·7	+4·1
178	M	3	12	4	12	24	...	28	·006	·002	...	210·1	+3·9
179	M	Dec. 5	15	Dec. 6	11	20	...	18	·006	·002	537	146·6	+0·1
		1894.		1894.									
180	V	Jan. 3	16	Jan. 4	14	22	...	52	·018	·003	538	124·0	−3·5
181	M	11	19·8	12	10	14	S	20	·012	·001	...	16·5	−4·4
182	A	Feb. 20	20·3	Feb. 22	6	34	S	36	·011	·004	540	209·6	−7·1
183	V	22	22·5	24	10	36	S	50	·015+	·009	...	182·3	−7·1
184	G	25	7·9	26	1	17	S	55	·021	·007+	...	150·5	−7·2
185	V	28	15·3	Mar. 1	3	12	S	50	·019	·005+	...	107·1	−7·2
186	M	Mar. 21	12	23	12	48	...	25	·005	·002	541	192·1	−6·9
187	V	30	17	Apr. 1	0	31	...	56	·018+	·011	...	70·6	−6·5
188	V	Apr. 17	13	18	11	22	...	42	·009	·004	542	195·2	−5·3
189	A	June 9	14	June 11	11	45	...	28	·014	·004	544	213·8	+0·6
190	A	July 2	2	July 3	2	24	...	38	·008	·002	545	275·9	+3·2

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		d	h	d	h	h						°	°
		1894.		1894.									
191	G	July 20	6·0	21	2	20	S	60	·036	·014	...	35·5	+5·0
192	G	Aug. 20	3·0	Aug. 20	18	15	S	65	·022+	·012+	547	347·3	+7·0
193	V	Sept. 14	13	Sept. 15	8	19	...	36	·016	·010	...	11·5	+7·2
194	M	19	12	21	6	42	...	25	·008	·004	548	306·1	+7·1
195	V	Nov. 13	14·1	Nov. 14	7	17	S	48	·020	·012	550	299·6	+2·8
		1895.		1895.									
196	M	Feb. 8	13	Feb. 8	22	9	...	23	·006	·002	553	234·0	−6·6
197	A	9	11	10	11	24	...	33	·009	·003	...	222·0	−6·7
198	M	15	14	16	10	20	...	26	·007	·002	...	141·3	−6·9
199	A	Mar. 8	14	Mar. 9	12	22	...	30	·006	·001	554	224·7	−7·3
200	A	13	13	15	12	47	...	35	·010	·003	...	159·3	−7·2
201	A	Apr. 11	6	Apr. 12	6	24	...	30	·010	·003	555	140·7	−5·8
202	A	May 10	7	May 11	1	18	...	32	·010	·002	556	117·1	−3·0
203	M	29	2·7	29	23	20	S	27	·008	·001	557	228·0	−0·9
204	M	Sept. 30	2	Sept. 30	23	21	...	24	·007	·002	...	28·9	+6·7
205	A	Oct. 12	16	Oct. 14	12	44	...	33	·006	·003	562	222·8	+6·0
206	A	Nov. 9	13	Nov. 13	0	83	...	37	·008	·004	563	215·2	+3·3
		1896.		1896									
207	A	Jan. 3	0	Jan. 6	0	72	...	36	·005	·001	565	217·7	−3·5
208	M	31	12	Feb. 1	12	24	...	23	·008	·001	566	202·4	−6·1
209	A	Feb. 4	12	5	1	13	...	24	·009	·002	...	149·7	−6·4
210	V	28	9	Mar. 1	0	39	...	42	·012	·003	567	195·4	−7·2
211	M	Mar. 4	7	5	5	22	...	29	·006	·003	...	130·6	−7·3
212	M	26	13	27	12	23	...	26	·006	·003	568	197·2	−6·7
213	V	May 2	13	May 4	3	38	...	44	·014	·007	569	68·6	−3·8
214	A	17	12	18	12	24	...	26	·009	·003	570	230·9	−2·1
215	M	July 11	15	July 12	12	21	...	22	·007	·002	572	221·3	+4·2
216	M	23	18·2	24	12	18	S	16	·005	·001	...	60·9	+5·3
217	A	Aug. 1	12	Aug. 2	12	24	...	24	·010	·002	573	305·2	+6·0
218	M	6	16	7	12	20	...	19	·007	·002	...	236·8	+6·3
219	M	29	16·6	30	11	18	S	14	·007	·001	574	292·6	+7·2
220	V	Sept. 18	1	Sept. 18	23	22	...	33	·015	·003	...	37·1	+7·1
221	A	20	3	21	0	21	...	38	·010	·002	...	9·6	+7·0
222	A	Oct. 11	13	Oct. 13	4	39	...	36	·012	·003	575	86·9	+6·0
223	A	Nov. 7	12	Nov. 8	12	24	...	35	·007	·003	576	91·5	+3·4
224	V	Dec. 3	17	Dec. 5	9	40	...	48	·007	·002	577	105·9	+0·3

	Class.	Period of Disturbance. Greenwich Civil Time. From		To		Duration in Hours.	Beginning.	Extreme Amplitude of Movements. Dec.	H.F.	V.F.	No. of Rotation.	Heliographic Coordinates of Centre of Sun's Disc. Longitude.	Latitude.
			d h		d h	h		′				°	°
5	A	1897. Jan.	2 12	1898. Jan.	3 3	15	...	38	·005	·004	578	73·5	−3·5
6	M	Feb.	25 15	Feb.	28 7	64	...	20	·005	·001	580	80·8	−7·2
7	M	Mar.	10 21	Mar.	11 3	6	...	23	·006	·001	581	266·2	−7·2
8	M	Apr.	1 18	Apr.	2 10	16	...	26	·010	·003	582	337·8	−6·4
9	A		20 12		21 3	15	...	30	·007	·004	...	90·3	−5·0
0	M		23 13		25 12	47	...	28	·008	·002	...	50·1	−4·7
1	M	May	20 21	May	21 17	20	...	20	·008	·002	583	48·7	−1·8
2	M	Sept.	4 13	Sept.	5 12	23	...	14	·005	·001	587	77·7	+7·3
3	M	Oct.	1 20	Oct.	3 11	39	...	22	·005	·001	588	77·5	+6·6
4	V	Dec.	11 3	Dec.	11 22	19	...	45	·007	·003	591	230·8	−0·7
5	V		20 13		21 21	32	...	49	·009	·004	...	106·6	−1·9
6	M	1898. Jan.	16 16	1898. Jan.	19 5	61	...	25	·006	·001	592	109·4	−4·9
7	M	Feb.	11 6	Feb.	12 3	21	...	28	·007	·002	593	132·6	−6·8
8	M		12 14		13 3	13	...	21	·004	·001	...	115·0	−6·8
9	M		14 13		15 11	22	...	20	·007	·001	...	89·2	−6·9
0	M	Mar.	11 14	Mar.	12 6	16	...	27	·008	·002	594	119·8	−7·2
1	G		15 0·9		16 8	31	S	75	·018+	·010+	...	73·9	−7·1
2	M	Apr.	12 16	Apr.	13 10	18	...	20	·006	·002	595	56·2	−5·7
3	M	Aug.	16 13	Aug.	17 11	22	...	22	·007	·002	600	191·2	+6·8
4	M	Sept.	2 14	Sept.	3 11	21	...	23	·006	·002	601	326·0	+7·2
5	G		9 14		10 11	21	...	55	·020+	·010	...	233·6	+7·2
6	A	Oct.	25 12	Oct.	26 12	24	...	32	·006	001	603	347·8	+4·8
7	M	Nov.	21 12	Nov.	22 12	24	...	20	·007	001	604	351·8	+1·9
8	M	1899. Jan.	28 18·8	1899. Jan.	29 9	14	S	26	·007	·003	606	172·2	−5·9
9	V	Feb.	11 22	Feb.	13 4	30	...	44	·007	·002	607	346·3	−6·8
0	M		23 12		24 11	23	...	22	·004	·001	...	193·7	−7·1
1	A	Mar.	21 22	Mar.	22 12	14	...	32	·008	·002	608	205·6	−6·9
2	M		23 15		24 4	13	...	29	·008	·002	...	183·0	−6·9
3	M	Apr.	18 12	Apr.	19 3	15	...	26	·006	·002	609	201·6	−5·2
4	M	May	1 14	May	2 11	21	...	20	·005	·002	...	28·7	−4·0
5	M		3 12		4 8	20	...	29	·007	·002	...	3·4	−3·8
6	M		15 13		16 7	18	...	29	·006	·002	610	204·1	−2·4
7	V	June	28 12	June	30 11	47	...	40	·010	·003	612	342·4	+2·8

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		d h 1899.	d h 1899.	h		′				°	°
258	M	Sept. 26 0	Sept. 27 0	24	...	20	·007	·001	615	239·3	+6·9
259	A	Oct. 23 18	Oct. 24 3	9	...	36	·006	·001	616	233·3	+5·0
		1900.	1900.								
260	M	Jan. 19 18	Jan. 20 0	6	...	28	·006	·001	619	153·7	−5·1
261	A	Mar. 13 2	Mar. 14 1	23	...	28	·009	·005	621	184·5	−7·2
262	A	May 5 3·2	May 6 0	21	S	29	012	·008	623	204·4	−3·6
		1901.	1901.								
263	M	Mar. 24 14	Mar. 25 3	13	...	23	·006	·002	635	255·4	−6·8
264	A	May 10 11	May 11 0	13	...	32	·007	·001	637	356·4	−3·1
265	M	Sept. 10 10	Sept. 11 5	19	...	24	·005	·001	639	170·0	+7·2
		1902.	1902.								
266	M	Jan. 15 19	Jan. 16 6	11	...	28	·005	·001	646	290·7	−4·7
267	A	Apr. 10 23	Apr. 11 15	16	...	28	·009	·001	649	248·5	−5·9
268	A	Aug. 21 13	Aug. 22 9	20	...	30	·004	·001	654	294·9	+7·0
		1903.	1903.								
269	M	Apr. 5 23·5	Apr. 6 18	18	S	26	·010	·003	663	176·7	−6·2
270	M	June 29 13	June 30 5	16	...	18	·006	·002	665	138·5	+2·8
271	M	Aug. 21 20	Aug. 23 0	28	...	28	·006	·001	667	153·6	+6·9
272	M	Sept. 19 16	Sept. 21 0	32	...	20	·004	·002	668	132·8	+7·1
273	A	Oct. 12 16	Oct. 14 4	36	...	37	·011	·004	669	189·3	+6·0
274	G	31 6·0	Nov. 1 5	23	S	121	·064	·032	670	304·1	+4·4
275	V	Dec. 13 13	Dec. 14 2	13	...	58	·008+	·008	671	93·5	−0·8
276	A	30 21	31 9	12	...	37	·005	·002	672	225·2	−2·9

2. *Attempted Solutions.*

Having formed Table I., the rather heavy work was next undertaken of examining the state of the Sun's surface at or near the time of commencement of each of the 276 disturbances tabulated, in the same manner as I had done for the "Nineteen Great Storms" in my former paper. This involved following up the history of more than 800 groups of spots, for usually there were several on the disc at the times in question; in one case there were

as many as 14. The general result from this part of the investigation was to confirm the conclusion to which a less extended examination on the same lines had led me more than twelve years ago—"though unusually large sun-spots are answered by unusually violent magnetic storms, we cannot as yet proceed further and express the magnitude or character of the magnetic disturbances in terms of the spotted area of the Sun, or of its principal groups at the time of observation." * The fact of a *general* correspondence between the numbers and areas of sun-spots on the one hand and the frequency and intensity of magnetic disturbances on the other was clear. An obvious correspondence between *individual* spots and storms was also manifest when both were of the first rank of magnitude ; but a few cases of apparent failure of correspondence were noted when those of the second rank were examined, and these failures became both absolutely and relatively more numerous the smaller the storms or sun-spots included in the inquiry. The two little tables given by the Rev. W. Sidgreaves at the end of his recently published paper, "On the Connexion between Solar Spots and Earth-magnetic Storms," † well represent the general result of this stage of my work. There was "a parallel progression of magnitudes," but there were also some failures of correspondence which seemed to be inconsistent with the conclusion to which the general accord appeared to point.

These failures on the one hand and the very numerous cases on the other, in which several important groups were on the Sun at the time, so that it was impossible to say which should be taken as related to the disturbance, or whether the latter should be ascribed to the joint effect of two or more of them, led me to try if some other class of solar phenomenon showed such a *general* accord with the magnetic disturbances as to warrant the belief that it would give a better particular and special accord than that shown by the spots. For this purpose I prepared Table II. It seemed unfair to give the same weight to a movement lasting over a very few hours as to one lasting several days, so each disturbance in Table I. has in the preparation of Table II. been reckoned according to the number of days or parts of a day during which it was in action. But if the movement attained a different degree of intensity on the different days through which it lasted it has been reckoned accordingly. Thus the "great" storm of 1882 November 17–21 has been reckoned as two days of "great" disturbance, one of "very active" and one of "active"—four days in all. The number of days of disturbance, therefore, is considerably greater than the number of separate outbursts. In Table II. account has also been taken of the "moderate" movements of 20′ and upwards, not shown in the Greenwich plates, but recorded in the notes. These were usually short-lived.

* *Knowledge*, vol. xv. 1892 May, p. 93.
† *Memoirs R.A.S.* vol. liv. pp. 95 and 96.

TABLE II.

Days of Magnetic Disturbance.

Year.	"Great."	"Very Active."	"Active."	"Moderate." In the Plates.	"Moderate." From the Notes.	"Moderate." Total.	Total.
1882	5	6	11	5	6	11	33
3	...	4	9	30	1	31	44
4	...	1	5	18	1	19	25
5	...	2	4	14	...	14	20
6	1	2	10	16	2	18	31
7	...	1	1	15	1	16	18
8	...	1	5	22	1	23	29
9	...	...	3	14	...	14	17
1890	...	...	1	8	...	8	9
1	...	2	11	14	...	14	27
2	6	11	4	21	3	24	45
3	...	2	7	16	8	24	33
4	3	7	3	9	8	17	30
5	...	...	10	6	6	12	22
6	...	4	10	10	3	13	27
7	...	2	2	12	1	13	17
8	2	...	1	12	1	13	16
9	...	2	2	11	...	11	15
1900	...	...	2	1	...	1	3
1	...	...	1	2	...	2	3
2	...	...	2	1	...	1	3
3	1	1	2	8	...	8	12

In the accompanying diagram, fig. 1, the first four lines exhibit the total number of days of disturbance above a certain amount in each year. The first line gives the "great" storms or movements above 60′ in declination; the second, all those above 40′—the "great" and "very active"; the third, those above 30′; the fourth, above 20′, no weighting being applied for the amplitude or intensity of the movement. But in the fifth line each separate class has been weighted so as to give practically the same total value to each of the four classes. The fifth curve, therefore, is practically the mean of the other four.

It will be seen that the years 1882 and 1892 are years of strongly marked maximum for each of the curves except the fourth unweighted one, the number of moderate disturbances being singularly small in 1882, which was very prolific of the greater storms. With this exception the salient features of all the curves are practically the same; the sharply defined maximum

of 1882 is followed by a prolonged minimum, broken only by a revival in 1886. The greater and more sharply defined maximum of 1892 is followed in like manner by a sudden drop, which proceeds in a succession of diminishing waves—1894, 1896, and 1898 being years of slight recovery. It will be seen that no material difference is made in these respects, however we treat the material before us, whether we weight the disturbances according to their

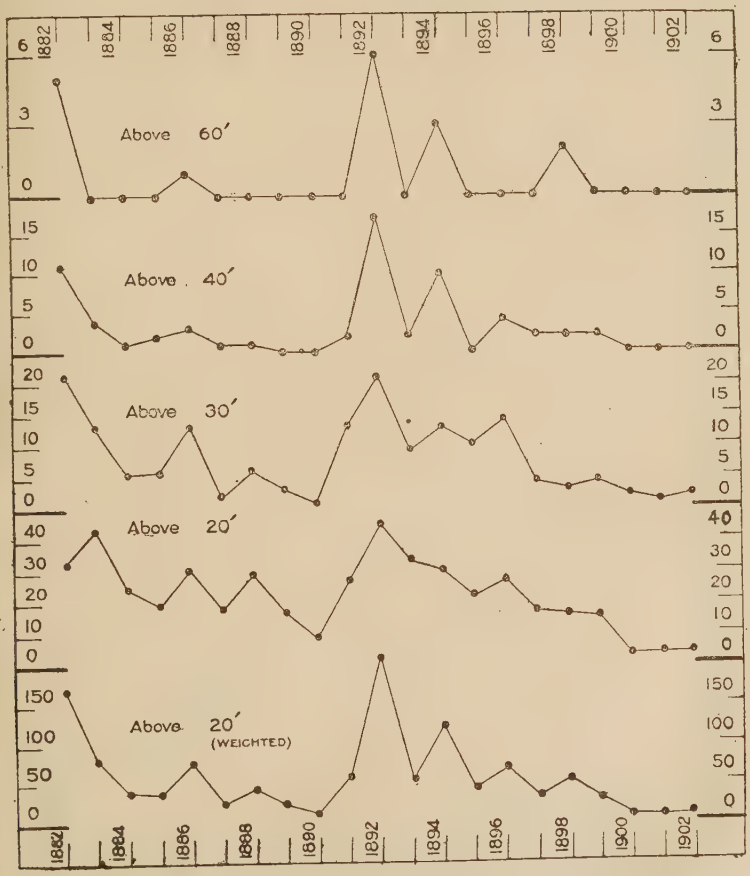

Fig. 1.—Curves of Frequency of Magnetic Disturbances, 1882 to 1903.

importance or whether we take them unweighted; whether we adopt so low a limit as 20′ of movement or whether we confine ourselves to those exceeding twice that amount.

Fig. 2 shows how the weighted curve compares with the various curves of solar activity. The prominence numbers represented are those given by Professor Riccò in vol. xxxii. of the

Memorie della Società degli Spettroscopisti Italiani, and they are shown in two forms. First the northern and southern hemi-

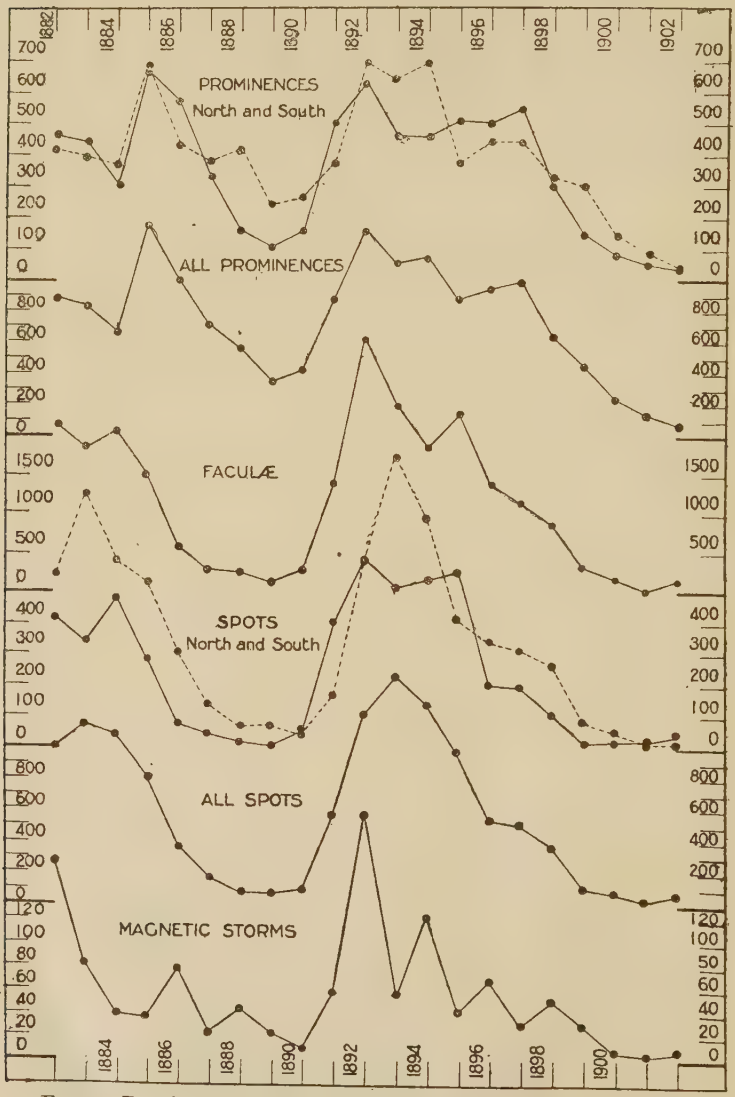

FIG. 2.—Prominences, Faculæ, and Sun-spots compared with Magnetic Disturbances, 1882 to 1903.

spheres are given separately, the northern numbers being joined by a continuous line, the southern by a broken one. Secondly,

the prominences for both hemispheres are given in one curve. Similarly in the fourth line the sun-spot numbers are shown separately for the northern and southern hemispheres, whilst in the fifth line they are combined. The numbers given for the spots and for the faculæ (shown in the third line) are taken from the Greenwich heliographic results.

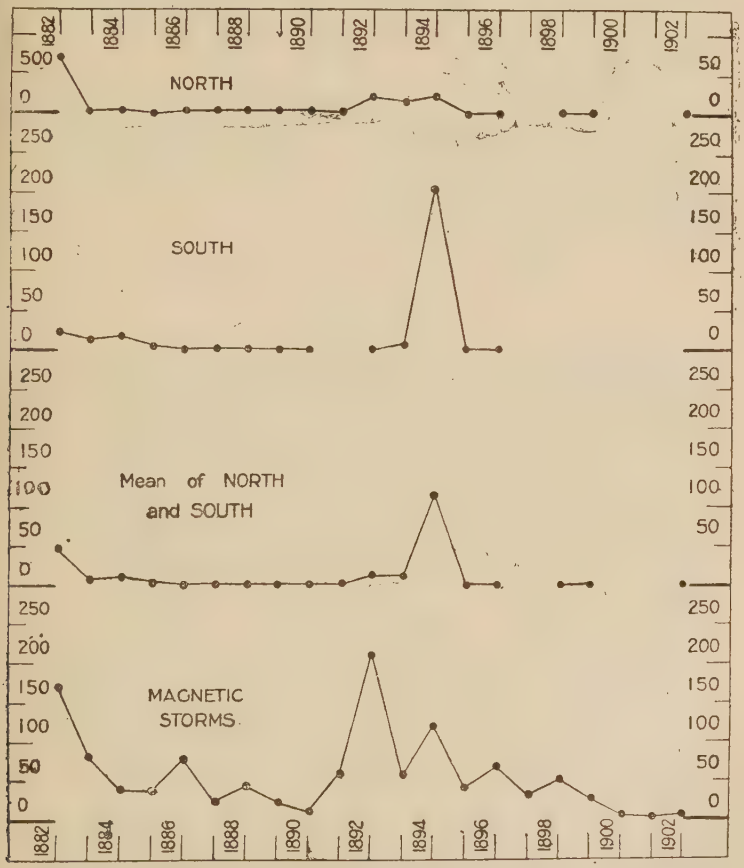

Fig. 3.—Prominences within 20° of the Sun's Pole compared with Magnetic Disturbances, 1882 to 1903.

It will be seen at once that, though the solar cycle shows itself broadly as a whole in the magnetic curve, the chief details of the magnetic curve are not all reproduced in any single one of the other curves. The accord is best with the curve of the faculæ, in which the maximum of 1892 is most strongly accentuated. But 1884 and 1895, both years of great faculous area, were very

quiet years magnetically ; and 1886 and 1894, years of magnetic activity, were years of falling off with the faculæ. The inference to be drawn would seem to be that there is no evidence at present of a closer connexion of either prominences or faculæ, rather than sun-spots, with magnetic disturbances sufficient to warrant us in discarding spots, which we can observe continuously, in favour of prominences or faculæ, the observations of which are necessarily defective just where it is most to be desired that they should be complete—that is to say, in the centre of the disc.

Fig. 3 exhibits the prominences of the Sun's polar regions—that is to say, within 20° of the Sun's pole—in comparison with the curve of magnetic disturbance ; for the suggestion has been made that it is the polar prominences that have the greatest effect upon terrestrial magnetism. The curves do not seem to lend any support to this suggestion.

3. *Demonstration of the Solar Origin of the Magnetic Disturbances.*

So far no relation of importance had been brought to light ; but a suggestion of another method of dealing with the problem arose from a further consideration of my former paper. Of the nineteen "great" magnetic storms sixteen synchronised with the presence of a large group near the centre of the disc. Three did not. But these three are from one point of view even more instructive than the other sixteen, for they synchronised with the *return* in a diminished form of a spot group which had once been large. Of the nineteen storms two were repetitions at the precise interval of a single rotation period of the Sun of two others of the series.

This circumstance suggested the addition to Table I. of the last three columns—viz. a column giving the number of the rotation of the Sun, and two columns giving respectively the longitude and latitude of the centre of the Sun's disc at the commencement of the storm. The same numeration of the rotations, the same prime meridian and rotation period, have been adopted as in the heliographic results given in the annual volumes of the Greenwich Observations.

Fig. 4, Plate 1, represents graphically the information given in columns 3, 5, 10, and 11 of Table I. A separate space in the horizontal direction is assigned to each rotation of the Sun, and each disturbance is represented by a straight line the length of which represents its duration, the line being so placed that its beginning is under the longitude corresponding to that of the centre of the Sun's disc at the time the storm began.

A mere inspection of the diagram brings out a striking and most important relation. The disturbances are not distributed irregularly with regard to the solar meridians, but chiefly affect two or three regions. More noticeable still is the frequency with

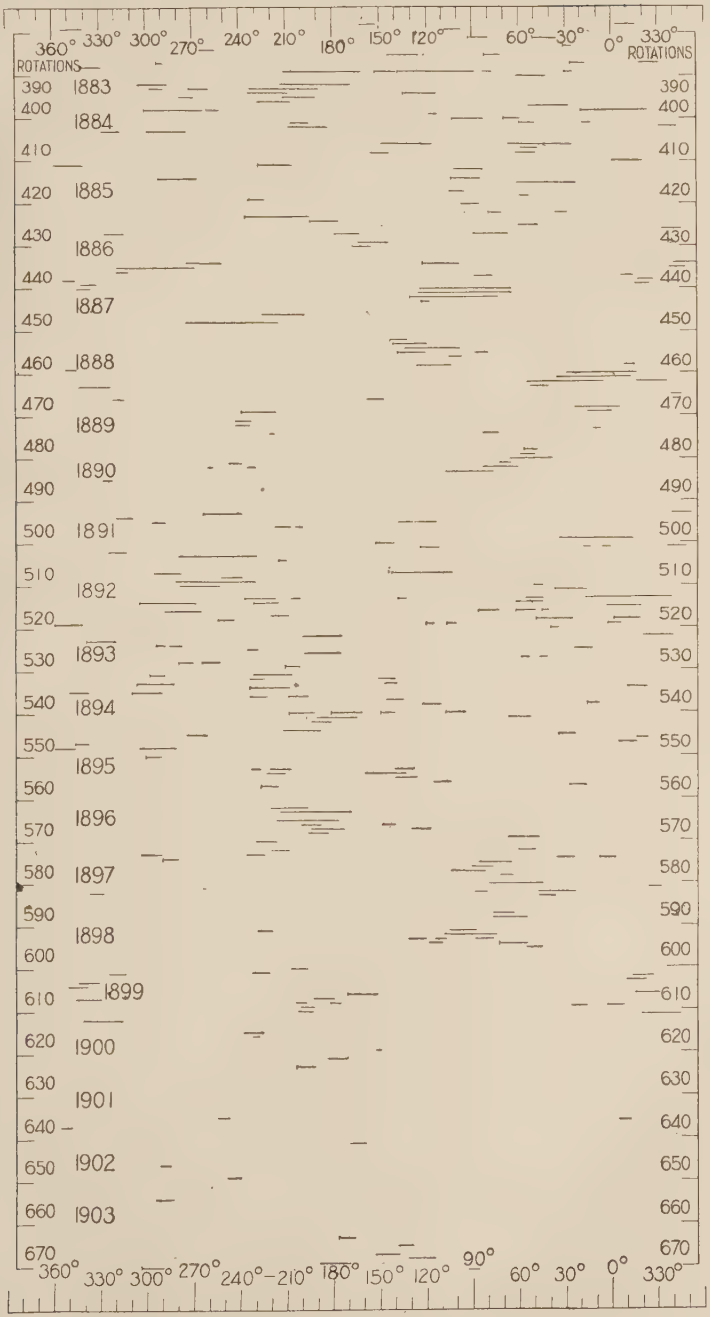

FIG. 4.—Distribution of Magnetic Disturbances, 1882 to 1903, according to the Heliographic Longitude of the Centre of the Sun's Disc at the time of their commencement.

which a disturbance in a given rotation is followed in the next rotation by another when the same meridian of the Sun has returned to the centre of the disc. Table III. shows that there are no fewer than thirty-six distinct sequences of this kind. Nor are these sequences always limited to a single return. The table shows that of the thirty-six sequences, whilst twenty-three extend only to two rotations, eight run through three, four through four, and one through six. It will further be seen that in a large number of cases (printed in heavier type in order to distinguish them) not only did a disturbance recur at the end of a complete rotation of the Sun, but no other disturbance had occurred in the interval.

It will be noted that though, for convenience, Carrington's rotation period has been assumed the remarkable sequences Nos. II., XXI., and XXIII. themselves independently supply a rotation period which does not differ materially from it; indeed, the two last named actually suggested the method of inquiry adopted.

TABLE III.

Magnetic Disturbances recurring in Consecutive Rotations

Reference No. of Sequence.	No. of Rotation.	Reference No. of Disturbance.	Class.	Coordinates of Centre of Sun's Disc.	
				Longitude.	Latitude.
				°	°
I.	519	151	G	359·9	+4·7
	520	154	G	362·8	+6·6
II.	438	72	A	353·9	+2·9
	439	73	V	343·5	+5·6
	440	74	M	345·4	+7·0
III.	603	246	A	347·8	+4·8
	604	247	M	351·8	+1·9
IV.	435	69	A	320·3	−5·8
	436	70	V	321·5	−3·2
V.	392	19	M	306·0	−5·7
	393	21	A	298·5	−7·1
VI.	573	217	A	305·2	+6·0
	574	219	M	292·6	+7·2
VII.	509	131	A	283·4	+5·1
	510	132	A	280·8	+2·0
VIII.	393	23	M	232·6	−7·2
	394	24	M	234·0	−6·7

Reference No. of Sequence.	No. of Rotation.	Reference No. of Disturbance.	Class.	Coordinates of Centre of Sun's Disc.	
				Longitude.	Latitude.
IX.	471	99	M	243°6	−2°4
	472	100	M	243·0	−5·3
X.	481	107	M	247·2	+7·0
	482	110	M	236·2	+5·3
XI.	513	135	G	238·2	−6·8
	514	141	G	233·2	−7·2
XII.	531	169	A	232·7	+1·7
	532	170	V	230·4	+4·6
XIII.	615	258	M	239·3	+6·9
	616	259	A	233·3	+5·0
XIV.	401	36	V	207·2	+6·4
	402	39	A	208·9	+4·2
XV.	553	197	A	222·0	−6·7
	554	199	A	224·7	−7·3
XVI.	562	205	A	222·8	+6·0
	563	206	A	215·2	+3·3
XVII.	566	208	M	202·4	−6·1
	567	210	V	195·4	−7·2
	568	212	M	197·2	−6·7
XVIII.	608	251	A	205·6	−6·9
	609	253	M	201·6	−5·2
	610	256	M	204·1	−2·4
XIX.	540	183	V	182·3	−7·1
	541	186	M	192·1	−6·9
	542	188	V	195·2	−5·3
XX.	429	65	M	164·3	+3·1
	430	66	M	167·8	−0·2
XXI.	452	81	M	143·4	+5·9
	453	82	M	141·7	+7·2
	454	83	V	134·2	+6·9
	455	84	M	138·5	+5·1

Reference No. of Sequence.	No. of Rotation.	Reference No. of Disturbance.	Class.	Coordinates of Centre of Sun's Disc.	
				Longitude.	Latitude.
				°	°
XXII.	532	171	M	155·4	+5·1
	533	173	A	146·9	+6·9
XXIII.	440	75	A	124·6	+7·2
	441	76	A	126·0	+6·3
	442	77	M	131·2	+4·1
	443	78	A	123·3	+0·7
XXIV.	399	33	M	119·5	+6·9
	400	34	V	103·7	+7·1
XXV.	591	235	V	106·6	−1·9
	592	236	M	109·4	−4·9
	593	238	M	115·0	−6·8
	594	240	M	119·8	−7·2
XXVI.	455	85	M	88·5	+4·8
	456	86	A	106·2	+1·9
XXVII.	575	222	A	86·9	+6·0
	576	223	A	91·5	+3·4
	577	224	V	105·9	+0·3
XXVIII.	587	232	M	77·7	+7·3
	588	233	M	77·5	+6·6
XXIX.	593	239	M	89·2	−6·9
	594	241	G	73·9	−7·1
	595	242	M	56·2	−5·7
XXX.	400	35	A	70·2	+7·1
	401	37	A	60·5	+5·7
XXXI.	406	43	M	67·0	−7·2
	407	44	M	58·0	−6·7
	408	46	M	63·8	−4·6
XXXII	478	104	A	57·0	+4·8
	479	105	M	60·0	+6·7
	480	106	M	66·5	+7·3
	481	108	M	73·4	+6·4
	482	111	A	83·4	+4·1
	483	112	A	108·8	+1·2

Reference No. of Sequence.	No. of Rotation.	Reference No. of Disturbance.	Class.	Coordinates of Centre of Sun's Disc.	
				Longitude.	Latitude.
				°	°
XXXIII.	513	138	A	56·9	−7·2
	514	142	M	63·2	−6·8
XXXIV.	582	230	M	50·1	−4·7
	583	231	M	48·7	−1·8
XXXV.	460	89	M	29·6	−7·1
	461	90	A	36·0	−5·7
	462	91	M	55·0	−3·3
	463	93	M	51·9	−0·1
XXXVI.	468	96	M	24·2	+5·3
	469	98	M	16·0	+2·4

There is no mistaking the conclusion enforced by this table. Covering as it does 295 rotations of the Sun, and including 276 disturbances, it is clear that on the average rather less than one disturbance will fall in each rotation. The probability, therefore, of any disturbance being succeeded by one in the next rotation within 10° of the same solar longitude is about one in forty. Yet forty-seven examples of this are given by Table III. But, as already noted, in several cases the same meridian is active for more than two rotations ; there are eight examples in which it runs to three, four in which it runs to four, and one in which it runs to six. The chances against these series being all accidental run into the millions. There is only one conclusion that we can draw from them—namely, that the magnetic disturbances which we experience here are intimately connected with the solar rotation period. Their exciting cause therefore lies in the Sun ; there is no possibility of ascribing them to some cause in planetary space, exciting at once the solar photosphere and terrestrial magnetism. Were it so the synodic rotation period of the Sun could not be brought out with such clearness by the times at which these disturbances occur.

This conclusion is quite independent of any comparison with either spots, faculæ, or prominences. If the Sun had never shown one single marking on disc or limb this conclusion would still be inevitable, provided only we knew the rotation period of the Sun; which the spectroscopic method might still have given us. It rests simply upon the times at which successive magnetic disturbances commence. If the Sun's surface were absolutely undiversified we should still know that certain restricted regions of it were capable of exercising this influence upon our Earth, although we had no other indication of any special action there taking place.

But when we examine more closely into the details of Table I. we find that there is a striking correspondence between the characteristics of these magnetic longitudes and those of the longitudes of spot-groups. If the only detail which we could fix concerning sun-spots was their solar longitude, three characteristics would yet make themselves apparent. First, not a few spot-groups would recur, showing themselves in a second rotation; but it would only be a small number that would make a third appearance, and appearances for a fourth or fifth time in succession would be quite rare. Next, different spot-groups would give different rotation periods. This of course is a result of the different rotation periods belonging to different latitudes, the rotation period at the equator being shorter than any other. Third, the frequency of intermittent groups would be remarked; a certain longitude would show a group of spots in one rotation, but would lie fallow during the next, to show a new outbreak in the third, or it might be in the fourth or fifth rotation.

It will be seen that precisely these three peculiarities are shown by the magnetic longitudes. As to the duration of a disturbance it has already been mentioned that we have twenty-three cases of a return in the second rotation, eight in the third, four in the fourth, and one in the sixth, a proportion which corresponds very well with the longevity of spot-groups of the more important and stable kind. The second point, that of the different rotation periods to be derived from these magnetic longitudes, is exceedingly important. In general the rotation period indicated does not differ widely from the mean period adopted in the Greenwich reductions—namely, 25·38 days, corresponding to a daily motion of 14° 11′. But sequences Nos. XXVII., XXXII., and XXXV. indicate a much more rapid rotation, and Nos. VI., X., XXIV., and XXIX. a much slower rotation. In the mean the former correspond to a daily motion of 14° 32′, 5′ larger than the mean motion found by Carrington for the equator, whilst the latter give a mean motion of 13° 39′, corresponding to the mean of those found by Carrington for latitude 30°. The rotation periods therefore given us by the magnetic disturbances not only agree in the mean with the rotation periods given by sun-spots, but the limits within which they vary are the same. We are therefore not only precluded from looking for the exciting cause in regions outside the Sun altogether, but we are restricted to his surface and to that portion of it rotating within these limits of velocity; in other words, to the principal sun-spot zones. If our rotation period of the Sun had been derived from its polar regions, none of these sequences, shown in Table III., would have held good.

The third feature, that of intermittent action, shown by spot-groups is a very striking one. Two instances, taken quite at random, one in the northern and one in the southern hemisphere—may suffice.

TABLE IV.

Example of an Intermittent Northern Spot-Group.

Rotation.	No. of Group.	Mean Long.	Mean Lat.
534	3191	50°·44	+12°·41
535	3232	53·13	+11·77
536	3272	56·27	+10·63
	Interval of 5 rotations.		
541	3460	53·44	+ 9·79
542	3492	54·85	+11·90
	Interval of 3 rotations.		
545	3629	56·44	+ 7·52
	Interval of 4 rotations.		
549	3777	51·37	+ 7·40
550	3824	52·67	+10·93

TABLE V.

Example of an Intermittent Southern Spot-Group.

Rotation.	No. of Group.	Mean Long.	Mean Lat.
394	985	171°·53	− 8°·23
	Interval of 3 rotations.		
397	1056	170·19	−10·21
398	1071	176·50	−10·93
399	1094	177·96	−10·25
	Interval of 2 rotations.		
401	1130	177·38	− 6·70
	Interval of 2 rotations.		
403	1190	171·28	− 7·82
404	1226	167·45	− 9·10

The magnetic disturbances afford a great number of instances of this intermittent action. It will be sufficient to give here a table of those cases in which the disturbances recur at intervals of two rotations. It should be noted that in five examples no disturbance has occurred in the interval. As in Table III. these instances are printed in heavier type.

TABLE VI.

Magnetic Disturbances recurring after two Rotations.

Reference No. of Sequence.	No. of Rotation.	Reference No. of Disturbance.	Class.	Coordinates of Centre of Sun's Disc.	
				Longitude.	Latitude.
XXXVII.	533	172	A	309°·3	+6°·3
	535	176	A	310·6	+6·7
XXXVIII.	**548**	**194**	**M**	**306·1**	**+7·1**
	550	**195**	**V**	**299·6**	**+2·8**
XXXIX.	393	22	V	273·4	−7·2
	395	26	M	279·0	−5·1
XL.	469	97	A	239·8	+4·3
	471	99*	M	243·6	−2·4
XLI.	532	170*	V	230·4	+4·6
	534	174	M	235·0	+7·3
	536	177	V	234·7	+4·1
XLII.	394	24*	M	234·0	−6·7
	396	28	M	228·6	−1·8
XLIII.	**563**	**206***	**A**	**215·2**	**+3·3**
	565	**207**	**A**	**217·7**	**−3·5**
XLIV.	406	42	M	148·8	−7·1
	408	45	M	156·2	−5·2
XLV.	553	198	M	141·3	−6·9
	555	201	A	140·7	−5·8
XLVI.	**379**	**3**	**V**	**107·8**	**−7·1**
	381	**4**	**G**	**93·0**	**−5·4**
XLVII.	412	49	M	102·6	+6·4
	414	51	A	104·4	+6·6

* These disturbances occur also in Table III.

Reference No. of Sequence.	No. of Rotation.	Reference No. of Disturbance.	Class.	Coordinates of Centre of Sun's Disc.	
				Longitude.	Latitude.
				°	°
XLVIII.	**578**	**225**	**A**	**73·5**	**−3·5**
	580	**226**	**M**	**80·8**	**−7·2**
	582	229	A	90·3	−5·0
XLIX.	514	142*	M	63·2	−6·8
	516	146	G	63·3	−2·0
L.	**381**	**5**	**G**	**51·2**	**−5·0**
	383	**6**	**M**	**30·9**	**+1·3**
LI.	516	147	M	45·9	−1·9
	518	150	V	49·8	+4·3
	520	155	M	39·9	+7·3
LII.	387	11	G	26·3	+6·6
	389	17	A	30·4	+1·3

Tables III. and VI., which bring out so strikingly the tendency of magnetic disturbances to recur when definite meridians of the Sun are on the centre of the disc, include between them 121* disturbances out of the 276 of Table I. : that is to say, very nearly half. If longer intervals of time than two rotations be taken a considerably greater number of sequences would be shown, and not a few of those given in Tables III. and VI. would be found to be continued at irregular intervals over considerable lengths of time. Rather more than three-fourths of the entire number of disturbances of Table I. appear to fall naturally into these more extended and irregular sequences. These are of course much less certain in their character than are those cases in which two successive disturbances occur when the same solar longitude has returned to the centre of the disc after the interval of a single rotation, and on the probabilities of the case some at least must be merely accidental. But that some at least of these more extended and irregular sequences are real, and not due to mere chance grouping, may, I think, be fairly inferred from such an instance as that presented by the two sequences of Table VII. In the period from 1895 February 9 to 1896 March 27, a period of fifty-nine weeks, sixteen disturbances were observed. Of these sixteen no fewer than nine synchronised with the return of one solar meridian to the centre of the disc, and three with the return of another. It seems contrary to all probability that so striking a relation as this should be merely one of accident throughout.

* Table III. contains ninety-one disturbances; Table VI., thirty-five; but five are common to the two tables.

TABLE VII.

Example of two Magnetic Meridians of long-continued Activity.

No. of Rotation.	Reference No. of Disturbance.	Class.	Coordinates of Centre of Sun's Disc. Longitude.	Latitude.
553	197	A	222°·0	−6°·7
554	199	A	224·7	−7·3
Interval of 3 rotations.				
557	203	M	228·0	−0·9
Interval of 5 rotations.				
562	205	A	228·8	+6·0
563	206	A	215·2	+3·3
Interval of 2 rotations.				
565	207	A	217·7	−3·5
566	208	M	202·4	−6·1
567	210	V	195·4	−7·2
568	212	M	197·2	−6·7
553	198	M	141·3	−6·9
Interval of 2 rotations.				
555	201	A	140·7	−5·8
Interval of 11 rotations.				
566	209	A	149·7	−6·4

When disturbances at a still greater distance apart are compared a further relationship appears to suggest itself, for there seems to be a marked tendency for a magnetic meridian to be active at the interval of a year, or, more strictly speaking, of thirteen or fourteen rotations. In the ordinary way this would be a very difficult relation to establish ; it would be quite impossible to do so at the solar maximum when disturbances are numerous. But it will be seen from Table I. that in the year 1900 only three disturbances were recorded, and that each one of these three agreed closely in longitude with a disturbance thirteen rotations earlier. Table I. supplies in all some twenty-six examples beside the three just mentioned in which a disturbance is followed at the end of thirteen or fourteen rotations by another within 10° of the same longitude. No doubt many of these are merely chance correspondences, but the number found is quite twice as large as would be expected if all were such, and we may therefore fairly conclude that there were at least fourteen or fifteen instances of the same area being magnetically active at the beginning and end of a period of twelve months. Such an area, in order to be in evidence in this way, must of course have a rotation period

differing very slightly, if at all, from the 25·38 days adopted in the computation of the longitudes.

It may be of some interest to point out here that several of the "Nineteen great Storms" discussed in my former paper would appear to be members of long irregular sequences. The first three of the nineteen storms do not come within the scope of the present paper, and four more, Nos. 6, 7, 15 and 16 of my former paper, appear to have been isolated disturbances. The sequences suggested for the remaining storms are given in Table VIII.

TABLE VIII.

Magnetic Storm.						Associated Spot-Group.				
Ref. No. of Storm.	Ref. No. in Table I.	Class.	No. of Rotation.	Coordinates of Centre of Sun. Longitude.	Coordinates of Centre of Sun. Latitude.	No. of Spot Group.	Mean Area.	Mean Coordinates of Spot Group. Longitude.	Mean Coordinates of Spot Group. Latitude.	Distance South of Centre.
				°	°			°	°	
	3	V	379	107·8	−7·1					
4	4	G	381	93·0	−5·4	726	833	93·0	−19·3	13·9
5	5	G	381	51·2	−5·0	729	1744	64·8	−28·5	23·5
	6	M	383	30·9	+1·3					
8	68	G	434	122·5	−6·5	1860	487	116·0	−16·8	10·3
	75	A	440	124·6	+7·2					
	76	A	441	126·0	+6·3					
	77	M	442	131·2	+4·1					
	78	A	443	123·3	+0·7					
9	135	G	513	238·2	−6·8	2421	2402	255·7	−28·3	21·5
10	141	G	514	233·2	−7·2	2440	386	250·3	−28·4	21·2
	160	M	525	236·8	−3·8					
	169	A	531	232·7	+1·7					
	170	V	532	230·4	+4·6					
	174	M	534	235·0	+7·3					
	177	V	536	234·7	+4·1					
11	146	G	516	63·3	−2·0	2515	742	84·3	−14·0	11·8
	150	V	518	49·8	+4·3					
	155	M	520	39·9	+7·3					
12	151	G	519	359·9	+4·7	2583	829	33·1	−31·3	37·6
13	154	G	520	362·8	+6·6	2615	277	16·5	−30·4	37·0
	171	M	532	155·4	+5·1					
	173	A	533	146·9	+6·9					
	179	M	537	146·6	+0·1					
14	184	G	540	150·5	−7·2	3412	1208	186·9	−32·0	24·9

Magnetic Storm.						Associated Spot-Group.				
Ref. No. of Storm.	Ref. No. in Table I.	Class.	No. of Rotation.	Coordinates of Centre of Sun. Longitude.	Coordinates of Centre of Sun. Latitude.	No. of Spot Group.	Mean Area.	Mean Coordinate of Spot Group. Longitude.	Mean Coordinate of Spot Group. Latitude.	Distance South of Centre.
				°	°			°	°	
	239	M	593	89·2	−6·9					
17	241	G	594	73·9	−7·1	4702	970	119·5	−13·2	6·0
	242	M	595	56·2	−5·7					
	234	V	591	230·8	−0·7					
18	245	G	601	233·6	+7·2	4781	1556	239·0	−12·1	19·3
	258	M	615	239 3	+6·9					
	259	A	616	233·3	+5·0					
	266	M	646	290·7	−4·7					
	268	A	654	294·9	+7·0					
19	274	G	670	304·1	+4·4	5098	470	298·1	−18·8	23·2

The last column in the preceding table gives for the spot-group supposed to be connected with the storm the least distance of its centre from the centre of the Sun's disc—its distance, that is, when in transit across the central meridian. In every case the spot-group was south of the centre of the Sun.

It will be seen, by reference to the detailed history of the spot-groups given in my former paper, that the sequences, if they indicate a continued action from the same area, prove that such areas can be magnetically active both before the spot has formed and after it has disappeared. A very clear case of the magnetic action continuing after the disappearance of the spot occurred during the minimum year 1889. On July 17 an "active" disturbance began with the characteristic sharp movement. It was followed in the next rotation by a "moderate" one at a slightly increased longitude; and so on, rotation after rotation, for six successive rotations. There was no possibility of confusing any of the members of this sequence—No. XXXII. of Table III.—with those of any other; they followed, rotation after rotation, without any intermission, and the only other disturbances—three in number—which took place during the five months that the sequence lasted occurred just half a rotation distant from them in time—in other words, when this side of the Sun was turned directly away from us.

The state of the Sun at the time when the first of these disturbances occurred was this: The largest group of the year—Group No. 2090, described in my "Note on the Sun-spots of 1889"*—was crossing the disc for the second time as Group No. 2092, and a new group of intermittent character, which in the following rotation became the third group of the year in

* *Monthly Notices*, vol. l. p. 369.

area, had just broken out in the same latitude as Group No. 2092, but preceding it by 48° of longitude. It is not possible to say whether we should correlate the magnetic disturbance with the considerable area of Group No. 2092, or with the first stages of the formation of Group No. 2093, or with the joint action of the two groups ; but in any case the synchronism of spot and disturbance is sufficiently striking. The two groups returned in the next rotation as Groups No. 2098 and No. 2099 ; and Group No. 2093 was represented for a couple of days by a very small group in the rotation following that, but the magnetic disturbances continued to return for three rotations after the disappearance of this last feeble remnant of the spot-groups.

The following conclusions appear to result from the facts now before us :

First. The origin of our magnetic disturbances lies in the Sun ; not in any body or bodies affecting both. This is clear from the manner in which those disturbances mark out the solar rotation period ; not the actual sidereal period, but the synodic period ; the period as it appears to us.

Second. The areas of the Sun giving rise to our magnetic disturbances are definite and restricted areas, as the definiteness with which certain longitudes are indicated proves. Our storms are not due to a general action or influence diffused over the whole solar surface.

Third. The region of the Sun, wherein these magnetically active areas are situated, rotates with the speed of the chief spot-bearing zones, viz. latitudes 0° to 30°.

Fourth. As shown in my former paper the greatest magnetic storms are clearly connected with great sun-spots ; the instances of synchronism between individual storms and individual spots being too numerous and precise to be accidental.

Fifth. These active areas on the Sun can, it would seem, be magnetically active before the visible formation of a spot-group ; they evidently can continue to be magnetically active after the spot-group has disappeared. It would appear, then, that spot-formation is an important phase of the activity of these areas, but that other phases of that activity can both precede and survive such spot-formation, just as faculæ both precede and survive spots.

Sixth. The influence proceeding from the Sun, whatever its character, does not act equally in all directions. It does not radiate like light or heat, but its action is confined to a definite and very restricted direction. This appears from a consideration of the characteristic "sharp" movement with which so many magnetic disturbances, and all the more violent of them, begin.* It would be possible to account for this sudden and instantaneous effect, instantaneous over all the earth, as the impact of a wave

* This "sharp" commencement is both actually and relatively more frequent at the times of the solar maximum, when "great" and "very active" storms are most abundant.

of energy radiating in all directions from the Sun as a centre, if such storms bore no relation to each other. It is not possible so to account for such an effect when it is followed by others exactly at the interval of one or more synodic rotation-periods of the Sun. Such a relation can only be explained by supposing that the earth has encountered, time after time, a definite stream, a stream which, continually supplied from one and the same area of the Sun's surface, appears to us, at our distance, to be rotating with the same speed as the area from which it rises.

Seventh. The average diameter of such streams may be roughly estimated from noting the time which an average storm lasts. Those in Table I. give an average duration of thirty hours, in which time the longitude of the centre of the Sun's disc appears to us to change by 16°·5. This would imply an average diameter for these stream-lines of 20° supposing them to be circular in section. An average stream-line will therefore occupy about $\frac{1}{130}$th part of the entire sphere, instead of the whole of it, as the magnetic wave from the Sun would do if it spread out equally in all directions.

Eighth. It follows, therefore, that, if sun-spots be the real seat of the energy giving rise to our earth-magnetic disturbances, the majority of them must fail to affect us. A similar conclusion results from comparing the numbers of magnetic disturbances and of spot-groups ; for whilst Table I. contains only 276 entries, the Greenwich sun-spot record for the same period gives more than 4,500 spot-groups, of which more than 600 might be classed as considerable, the least important having been visible for at least eight consecutive days, and having a mean area of 200 millionths of the Sun's visible hemisphere.

Ninth. It follows from the fifth and eighth conclusions that, though sun-spots and magnetic disturbances are intimately connected, large sun-spots will often be observed when no disturbances are experienced, whilst sometimes disturbances will be experienced when no spots with which they can be associated are visible. The familiar and oft-repeated phenomenon of "intermittent spot-activity" suggests that often, if not always, the spot should be regarded in these cases as dormant rather than as having ceased to exist, the spot-forming forces being possibly still at work below the photosphere.

Tenth. The last column of Table VIII. suggests that stream-lines proceeding from the Sun and giving rise to the magnetic disturbances are not necessarily always truly radial in direction.

In the valuable paper by the Rev. Walter Sidgreaves already referred to, "On the Connection between Solar Spots and Earth Magnetic Storms," an immense amount of material has been discussed, and the results, as the author expressly remarks, afford proof of a real connection between spots and magnetic storms. He was, however, held back from the natural conclusion that the cause of these storms resided in the Sun by two considerations, the one observational, the other theoretical. The

observational difficulty was the fact to which I called special attention more than twelve years ago *—that great spots have been seen when there have been no storms, and storms experienced when there have been few or no spots. That difficulty is now removed, since it is seen that spots ought not always to be accompanied by storms on the one hand, whilst the storms themselves show their solar origin apart from any question of individual spots on the other.

On the theoretical difficulty Father Sidgreaves wrote: "The question, 'Is the source of energy affecting our magnets on the Sun?' is a question admittedly settled in the negative theoretically," and he quoted the well known presidential address of Lord Kelvin to the Royal Society in 1892. But Father Sidgreaves strangely passed over without notice a most significant qualification in Lord Kelvin's conclusion. Lord Kelvin wrote: "Thus in the eight hours of a not very severe storm as much work must have been done by the Sun *in sending magnetic waves out in all directions through space* as he actually does in four months of his regular heat and light." I have italicised certain words because these form the basis of Lord Kelvin's computation, and it is to these that his conclusion applies. It is only as we assume this condition, so explicitly stated by Lord Kelvin, that we can reach his conclusion. And that condition does not hold good. As I have shown in this paper the magnetic disturbances themselves supply absolutely conclusive evidence that they are not due to magnetic waves spreading out from the Sun equally in all directions through space. They are due to action along definite restricted lines.

There is no necessity for me to expound at length the magnitude of the change thus made in our way of regarding the solar action. The difference between the universal action of a "polarised magnetic sphere" and the action of restricted stream-lines is fundamental.

Stream-lines proceeding from the Sun have been actually photographed. In 1898, after the eclipse of that year, my wife and I wrote of the photographs taken by her: "The chief features shown by these long-exposure photographs are four long rays. . . . The lengths given for the rays are, of course, their apparent lengths; their real lengths are probably considerably greater, for we do not know in what plane they lie, nor how far their apparent lengths have been diminished by fore-shortening; the values given above" (13·9 lunar radii in the extreme case) "therefore are a minimum. The rays in appearance are straight, narrow, and rod-like up to the limits given." †

This was the first occasion upon which these rod-like rays were clearly photographed. The present paper, by an entirely different class of evidence, has shown that stream-lines analogous in form are being driven off from the Sun. The same photo-

* *Knowledge*, vol. xv. May 1892, p. 93. † *The Indian Eclipse*, p. 117.

graphs showed also for the first time the real significance of the synclinal structures of Mr. Ranyard. We wrote: "But their bases" (*i.e.* of the long rays) "are of an altogether different form. Each one rises from one of those 'synclinal structures' to which Mr. Ranyard called attention in his great eclipse volume (*Memoirs R. A. S.* vol. xli.) Only four of these structures were seen in this eclipse, and in each case we now see from these photographs that they terminate in one of these rod-like rays. The bending towards each other of these synclinal curves is therefore, not apparent only, as being due to some effect of perspective, nor accidental, but is of the very nature of their structure." The building up of these synclinal structures was shown in the same eclipse on photographs taken by Mr. C. Thwaites with the fine photo-visual telescope lent to him by Mr. G. J. Newbegin. Concerning these we wrote: "These show us that over the principal prominences and at some little distance an arch of coronal matter is formed. This is succeeded by a larger arch outside, and so on for a succession, the outer arches being less definite and complete than the inner ones. Outside all we find the curves defining the boundaries of the synclinal group. . . . From the apex of the synclinal structure we find the coronal matter driven outwards in a straight line, which probably indicates an immense velocity. It must be noted that this eruptive action is not always radial. One of the long rays in 1898 was tangential and another was oblique."*

As to the physical cause of these streams and the condition of the matter composing them it does not lie within my province to offer any suggestion. The one supplied by Professor Svante Arrhenius, published in the appendix to the *Monthly Notices*, vol. lxiv. No. 8, that they consist of minute droplets formed by condensation in the Sun's atmosphere, negatively charged and driven away by the pressure of radiation, seems entirely consistent with the appearance of the coronal photographs, and with the conditions indicated by the magnetic storms.

That, therefore, which Lord Kelvin spoke of twelve years ago as "the fifty years' outstanding difficulty" is now rendered clear. Our magnetic disturbances have their origin in the Sun. The solar action which gives rise to them does not act equally in all directions, but along narrow, well defined streams, not necessarily truly radial. These streams rise from active areas of limited extent. These active areas are not only the source of our magnetic disturbances but are also the seats of the formation of sun-spots, and their activity is ordinarily most easily and continuously manifested to us by the presence of sun-spots, and by the changes which such spots undergo. But these areas can be magnetically active both before a spot has formed and after it has disappeared. Though, therefore, sun-spots and magnetic

* *The Indian Eclipse*, p. 121.

disturbances have an ultimate connection the latter can occur when no spots are visible. On the other hand, since the solar action is restricted in its direction, many great spots may be visible to us without any effect being produced on the Earth's magnetism. But that the disturbances have an intimate connection with the spots is clear from the fact that they occur at intervals corresponding to the rotation period of the Sun as determined by sun-spots, and to the special rotation periods of those zones of the Sun where sun-spots most congregate, whilst they exhibit in the times of their returns some of the chief sun-spot characteristics, and in not a few instances individual storms have been clearly associated with individual groups of sun-spots.

Postscript.—It will be noted that I have rigidly confined myself to the disturbances which I scheduled in Table I. at the beginning of my inquiry. In not a few cases disturbances of slightly smaller amplitude tend to complete the less regular sequences, but I thought it better to omit all references to them rather than to enlarge indefinitely the scope of the paper.

86 *Tyrwhitt Road, St. John's, Brockley, S.E.:*
1904 *November* 4.

A Discussion of the Long-Period Terms in the Moon's Longitude.

By P. H. Cowell.

The following empirical corrections satisfy the observations of the Moon's longitude from 1750 to 1901 very closely, and, as far as I know, they are not inconsistent with the observations previous to 1750. The time is measured in units of 40×400 lunar days, or forty-five years nearly from the middle of my sixty-seventh period of analysis, or the year 1826·1.

The second line of the formula, which is supposed to be applied to the Hansen-Newcomb system at present used by the *Nautical Almanac*, is merely a removal of Newcomb's empirical term :

$$\begin{aligned} -4''.3 + 15''.0 \cos\ & 45°t - 9''.7 \sin\ 45°t \\ -12''.2 \cos\ & 60°t + 9''.5 \sin\ 60°t \\ -\ 2''.3 \cos & 246°t + 0''.5 \sin 246°t \\ +\ 0''.2 \cos & 390°t - 0''.7 \sin 390°t \end{aligned}$$

The period of the term $45°t$ is about 363 years ; that of Newcomb's term, 273 years ; the periods of the two shorter terms are 66 and 42 years.

The above formula has been formed on the assumption that

one, and only one, empirical term of very long period is to be introduced, and also that the secular acceleration is not to be altered. With these limitations I believe the solution is unique—that is to say, it only admits of variations proportional to the errors of observation, and no totally different solution can be found. The assumption of two or more long-period terms opens possibilities too vast to be discussed. As to the secular acceleration, the terms

$$+15''{\cdot}0 \cos 45°t - 9''{\cdot}7 \sin 45°t$$

in the above formula may be replaced by

$$+9''{\cdot}8 \cos 50°t + 5''{\cdot}2 - 4''{\cdot}4T^2$$
$$-7''{\cdot}2 \sin 50°t - 3''{\cdot}0T$$

where T is measured in centuries.

The argument $50°t$ has a period of 327 years, and, as the eclipses discussed by Professor Newcomb and Mr. Nevill (*Monthly Notices*, vol. xxxix. p. 73) point to a negative correction to the secular acceleration of about 2″, my conclusion is that if a single long-period empirical term alone exists its period should be about 350 years, and not 273 years, as Professor Newcomb has taken it.

Table I. gives the separate terms of the empirical formula at the beginning of this paper, tabulated for $t = -4$, -3, -2, *i.e.* for 180, 135, 90 years before 1826, and also for the middle of every fourth of my periods of analysis of 400 lunar days each from -1 to $+135$.

It will be seen that from 1650 to 1750 the numerical value of my formula is less than 5″. If, therefore, Professor Newcomb's empirical term satisfies the observations for these hundred years, so also does mine.

Table I.

Numerical Values of Empirical Terms. Unit 0″·1.

	$-43 + 150\cos 45°t - 97\sin 45°t$	$-122\cos 60°t + 95\sin 60°t$	Sum of Two Preceding Terms.	$-23\cos 246°t + 5\sin 246°t$	$+2\cos 390°t - 7\sin 390°t$	Sum of all Four Terms
$t = -4$	−193	+143	−50	...	...	...
$t = -3$	− 80	+122	+42	...	...	...
$t = -2$	+ 54	− 21	+33	...	...	...
Period						
− 1	+ 87	− 68	+19	−16	−5	− 2
+ 3	+ 95	− 81	+14	−22	−7	−15
7	+104	− 95	+ 9	−24	−6	−21
11	+111	−107	+ 4	−21	−3	−20
15	+118	−118	0	−15	+2	−13
19	+123	−128	− 5	− 5	+6	− 4

Period	$-43 + 150\cos 45°t - 97\sin 45°t$	$-122\cos 60°t + 95\sin 60°t$	Sum of Two Preceding Terms.	$-23\cos 246°t + 5\sin 246°t$	$+2\cos 390°t - 7\sin 390°t$	Sum of all Four Terms
23	+128	−137	− 9	+ 5	+8	+ 4
27	+132	−143	−11	+14	+6	+ 9
31	+134	−149	−15	+20	+1	+ 6
35	+135	−153	−18	+23	−4	+ 1
39	+136	−155	−19	+22	−7	− 4
43	+135	−155	−20	+16	−7	−11
47	+133	−154	−21	+ 9	−4	−16
51	+130	−151	−21	− 2	+1	−22
55	+126	−145	−19	−11	+5	−25
59	+120	−139	−19	−19	+7	−31
63	+115	−131	−16	−23	+6	−33
67	+107	−122	−15	−23	+2	−36
71	+ 99	−111	−12	−19	−2	−33
75	+ 90	− 99	− 9	−11	−7	−27
79	+ 80	− 87	− 7	− 1	−7	−15
83	+ 70	− 73	− 3	+ 8	−5	0
87	+ 59	− 58	+ 1	+17	0	+18
91	+ 47	− 43	+ 4	+22	+5	+31
95	+ 34	− 27	+ 7	+24	+7	+38
99	+ 21	− 11	+10	+21	+6	+37
103	+ 8	+ 5	+13	+14	+3	+30
107	− 6	+ 21	+15	+ 4	−2	+17
111	− 20	+ 37	+17	− 5	−6	+ 6
115	− 33	+ 52	+19	−15	−8	− 4
119	− 48	+ 68	+20	−21	−6	− 7
123	− 61	+ 81	+20	−23	−1	− 4
127	− 76	+ 95	+19	−22	+4	+ 1
131	− 89	+107	+18	−16	+7	+ 9
135	−103	+118	+15	− 8	+7	+14

Table II. gives, for each period from 1 to 89, (i) the mean error, tabular *minus* observed, of the Moon's longitude, the tabular places being those used by Airy with the corrections given in *Monthly Notices*, vol. lxiv. pp. 571–573; (ii) a long-period correction, representing the excess of the mean longitude and 273-year term now used in the *Nautical Almanac* over the mean longitude used by Airy; (iii) the sum of the two preceding columns.

The long-period terms of the Hansen-Newcomb tables are

$$+15''\cdot34 \sin (A + 30° 12')$$

as given by Hansen, and

$$-15''\cdot5 \cos A,$$

Newcomb's empirical term, where A is an argument whose numerical value for the middle of my pth period is (see *Monthly Notices*, vol. lxiv. p. 421)

$$183^\circ\cdot8946+1^\circ\cdot4953\ (p-44).$$

To get the mean longitude of the Hansen-Newcomb system now used in the *Nautical Almanac* I have formed $g+\omega-\odot$ from Hansen's *Tables de la Lune*, p. 15, and added in

$$-1''\cdot14-29''\cdot17\ T-3''\cdot76\ T^2$$

from Newcomb's corrections. The result is

$$335^\circ\ 43'\ 25''\cdot56+(1336^{rev}+1108390''\cdot44)\ T+9''\cdot541\ T^2 +0''\cdot013473\ T^3,$$

T being measured in Julian centuries from 1800 January 0·0 G.M.T.

Airy, copying from Damoiseau, 1824, uses

$$111^\circ\ 36'\ 42''\cdot8+(1336^{rev}+307^\circ\ 52'\ 41''\cdot6)\ T+10''\cdot7232\ T^2 +0''\cdot019361\ T^3,$$

the epoch being 1801 January $0^d\cdot5$ Paris Mean Time, which Airy takes as being $9^m\ 21^s\cdot5$ in advance of Greenwich Mean Time.

The excess, therefore, of the Hansen-Newcomb mean longitude over the Airy-Damoiseau is

$$-3''\cdot14+28''\cdot63\ T-1''\cdot1828\ T^2-0''\cdot005888\ T^3.$$

I neglect the cube term and transform the other terms into

$$-3''\cdot14+0''\cdot3246\ (p-44)-0''\cdot0001518\ (p-44)^2$$

for the middle of my pth period. The precise definition of my periods is given at the top of page 421, vol. lxiv.

Table III. gives for each period from 86 to 133 the mean error, tabular *minus* observed, of the Moon's longitude, the tabular places being based on Hansen's tables, but modified as described in vol. lxiv. pp. 85, 414, 415.

Periods 86–89 (years 1847 to 1851) form part of both tables, and to call attention to this I have inclosed the errors for these periods in brackets. It will be noticed that the two sets of errors for these four periods do not agree. This is partly due to differences in the tabular places depending on such arguments as ☌, 2M—E, $\omega-\omega'$, &c., but I have not succeeded in reconciling the two sets of figures completely.

TABLE II.

Errors, Tabular minus *Observed, of Moon's Longitude. Unit* 0″·1.

Periods 1–89.

Period.	App. Error of Airy's Tab. Places corrected as on lxiv. pp. 571–3.	Long-period Terms.	Sum of Preceding Columns or Corrected Apparent Error.	Period.	App. Error of Airy's Tab. Places corrected as on lxiv. pp. 571–3.	Long-period Terms.	Sum of Preceding Columns or Corrected Apparent Error.
1	+54	−18	+36	46	−10	+37	+27
2	+20	−15	+ 5	47	−18	+37	+19
3	+24	−12	+12	48	− 8	+36	+28
4	+23	− 9	+14	49	− 2	+36	+34
5	+ 7	− 6	+ 1	50	− 9	+35	+26
6	+13	− 3	+10	51	− 6	+34	+28
7	+12	0	+12	52	−24	+34	+10
8	+17	+ 3	+20	53	−27	+33	+ 6
9	+11	+ 5	+16	54	−11	+32	+21
10	+15	+ 8	+23	55	−15	+31	+16
11	+11	+10	+21	56	− 9	+31	+22
12	+21	+13	+34	57	+17	+30	+47
13	−11	+15	+ 4	58	+ 9	+29	+38
14	+ 8	+17	+25	59	+18	+28	+46
15	+ 6	+19	+25	60	+11	+27	+38
16	− 3	+21	+18	61	+25	+27	+52
17	−20	+23	+ 3	62	+19	+26	+45
18	−32	+24	− 8	63	+ 8	+25	+33
19	−14	+26	+12	64	+12	+24	+36
20	−10	+27	+17	65	+11	+23	+34
21	−29	+29	0	66	+ 9	+22	+31
22	−38	+31	− 7	67	+ 8	+21	+29
23	−48	+31	−17	68	− 1	+21	+20
24	−38	+32	− 6	69	+ 5	+20	+25
25	−42	+33	− 9	70	− 7	+19	+12
26	−61	+34	−27	71	− 1	+18	+17
27	−36	+35	− 1	72	+16	+17	+33
28	−23	+36	+13	73	+28	+17	+45
29	−40	+37	− 3	74	+22	+16	+38
30	−32	+37	+ 5	75	+19	+15	+34
31	−29	+38	+ 9	76	+11	+15	+26
32	−32	+38	+ 6	77	+10	+14	+24
33	−29	+39	+10	78	+ 8	+14	+22
34	−50	+39	−11	79	− 2	+13	+11
35	−44	+39	− 5	80	+ 9	+13	+22
36	−54	+39	−15	81	+ 6	+12	+18
37	−53	+39	−14	82	0	+12	+12
38	−56	+39	−17	83	− 5	+11	+ 6
39	−62	+39	−23	84	−21	+11	−10
40	−50	+39	−11	85	−12	+11	− 1
41	−15	+39	+24	86	(−17)	+11	(− 6)
42	−18	+39	+21	87	(−31)	+10	(−21)
43	−21	+38	+17	88	(−26)	+10	(−16)
44	−14	+38	+24	89	(−17)	+10	(− 7)
45	−18	+38	+20				

TABLE III.

Errors, Tabular minus *Observed, of Moon's Longitude. Unit* 0″·1.

Periods 86–133.

Period.	85 +	101 +	117 +
+1	(+ 6)	−30	+ 5
2	(− 4)	−18	+ 2
3	(−12)	−10	+ 7
4	(−18)	−17	+ 9
5	−23	−14	+10
6	−26	+ 1	+ 4
7	−29	+ 2	+ 7
8	−26	+ 3	+14
9	−33	+ 2	+10
10	−43	+ 2	0
11	−47	+ 3	− 4
12	−53	−10	−10
13	−63	−13	−18
14	−58	− 8	−18
15	−50	−10	−15
16	−37	+ 1	−19

Table IV. gives the sum of my empirical formula and the last column of Table II. Table V. gives the sum of my empirical formula and Table III. Tables IV. and V. are arranged in columns of sixteen periods to exhibit the fact that the errors still outstanding seem to be principally errors of comparatively short period, depending on such arguments as ☌, $2\varpi - 2J$, $2M - E$, &c. The change of phase between Tables IV. and V. is due to the large difference in the figure of Earth terms (argument ☌) in the two sets of tabular places.

TABLE IV.

Errors for each Period of Analysis corrected for Empirical Terms. 1750–1851. *Unit* 0″·1.

Period.	0. +	16. +	32. +	48. +	64. +	80. +
+1	+28	− 5	+14	+15	0	+10
2	− 7	−14	− 9	+ 6	− 4	+ 8
3	− 3	+ 8	− 4	+ 6	− 7	+ 6
4	− 2	+15	−15	−13	−16	− 6
5	−17	0	−16	−18	− 9	+ 8
6	−10	− 5	−20	− 3	−22	(+ 8)

Period.	0. +	16. +	32. +	48. +	64. +	80. +
7	− 9	−13	−27	− 9	−16	(− 3)
8	− 1	− 1	−17	− 4	+ 1	(+ 5)
9	− 4	− 3	+16	+19	+15	(+17)
10	+ 3	−19	+12	+ 8	+10	...
11	+ 1	+ 8	+ 6	+15	+ 7	...
12	+16	+21	+12	+ 6	+ 2	...
13	−12	+ 5	+ 6	+20	+ 3	..
14	+10	+12	+12	+13	+ 4	...
15	+12	+15	+ 3	0	− 4	...
16	+ 7	+11	+10	+ 2	+11	...

TABLE V.

Errors for each Period of Analysis corrected for Empirical Terms. 1847–1901.
Unit 0″.1.

Period.	85. +	101. +	117. +
+1	(+20)	+ 2	− 1
2	(+14)	+12	− 5
3	(+ 9)	+17	+ 1
4	(+ 6)	+ 7	+ 3
5	+ 5	+ 6	+ 5
6	+ 5	+18	0
7	+ 4	+16	+ 4
8	+ 8	+15	+12
9	+ 3	+11	+10
10	− 5	+ 8	+ 1
11	− 9	+ 6	− 1
12	−15	+ 4	− 5
13	−26	−12	−11
14	−21	−17	− 9
15	−15	−13	− 5
16	− 3	− 5	− 7

I now come to the analysis. The mean of the two sets of values for periods 86—89 has been taken.

To get rid of errors depending on $\mathfrak{S}$, $2\varpi - 2\mathrm{J}$, $2\mathrm{M} - \mathrm{E}$, $\omega - \omega'$, &c., means of seventeen consecutive periods have been taken for every fourth period from 11 to 123 inclusive. For periods 7 and 127 it is clearly only possible to take the mean of thirteen consecutive periods.

TABLE VI.

Mean Error, Tabular minus *Observed, of Moon's Longitude.* Smoothed.
Unit 0″·1.

Period.	0. +	32. +	64. +	96. +
+3	...	0	+33	−33
7	+16	+ 5	+28	−25
11	+14	+11	+23	−15
15	+12	+17	+17	− 5
19	+ 5	+25	+ 4	+ 1
23	+ 1	+30	− 9	+ 2
27	− 1	+31	−26	0
31	− 6	+32	−34	− 2

It is these thirty-one numbers in Table VI. that form the subject of the following analysis. It is required to find an empirical formula for these observed quantities.

Now at the outset every element of an empirical term is arbitrary, coefficient, period, and phase. It is at any rate possible to avoid uncertainties of phase in the following way. I add and subtract the errors equidistant from the middle or period 67. The added errors (ϵ_t) must be satisfied by an empirical formula consisting of cosines only; the subtracted errors (ε_t') by sines only. As I have not divided by 2 the unit is now 0″·05. The time is now measured from period 67 (or 1826·1) in units of forty periods, or forty-five years.

TABLE VII.

Values of ϵ_t. *Unit* 0″·05.

tot.	ϵ_t	tot.	ϵ_t	tot.	ϵ_t	tot.	ϵ_t
0	+66	4	+29	8	−33	12	+ 6
1	+60	5	+ 8	9	−31	13	+14
2	+54	6	−15	10	−16	14	+14
3	+47	7	−29	11	− 4	15	+14

TABLE VIII.

Values of ϵ_t'. *Unit* 0″·05.

tot.	ϵ_t'	tot.	ϵ_t'	tot.	ϵ_t'	tot.	ϵ_t'
		4	−21	8	−33	12	− 4
1	− 4	5	−26	9	−19	13	−10
2	− 8	6	−37	10	−14	14	−14
3	−13	7	−39	11	− 6	15	−18

The ranges in the two tables are respectively 99 and 39. It is clearly better therefore to begin with the Even Function Analysis.

When a period is known, and its coefficient alone is required, the argument being at, it is usual to multiply each error by $\cos at$ (or $\sin at$, as the case may be), and deduce the coefficient from the sum of the products

$$\Sigma\epsilon \cos at.$$

Now suppose a term $b \cos \beta t$ added to the tabular places, and therefore to each quantity ϵ_t. The above expression then receives the increment

$$b\Sigma \cos at \cos \beta t$$

and the method breaks down unless there is reason to suppose $\Sigma \cos at \cos \beta t$ is zero, or at any rate small.

Now when a is very large and the analysis extends over many hundred periods of at, and at least one complete period of $(a-\beta)t$, the process is unobjectionable. Even then, as I have pointed out in vol. lxiv. p. 413,

$$\Sigma \sin \theta \sin (\theta \pm \mathrm{D})$$

is not zero when the summation extends over observations for which D is never zero, and for which $\cos \mathrm{D}$ averages $-\frac{1}{2}$; but an approximate allowance can be made in such cases.

When, however, a is very small, and we have only two periods of at, extreme caution is required.

Supposing, for instance, the coefficient of $\sin at$ is required from observations that extend over two revolutions of the argument from $at = -360°$ to $at = +360°$, and a term $5'' \sin \frac{1}{4}at$ exists in the errors.

If the coefficient of $\sin at$ be taken as the mean value of $2\epsilon \sin at$, the result is in error by $10'' \times$ mean value of $\sin at \sin \frac{1}{4}at$, which works out to about $2''$.

Now this illustration is very much in point, for I am trying to prove the existence of a 66-year term which goes through two revolutions in 150 years, and Newcomb's empirical term corresponds closely to the term $\frac{1}{4}at$, a term with four times the period. Also I believe that Newcomb's term should be replaced by one with a period 30 per cent. longer. My estimate of the errors of Newcomb's term is given in the fourth column of Table I., but the possibility of even larger errors must be borne in mind. A maximum error of $5''$ may, in certain circumstances as to phase, cause an error of $2''$ in the deduced coefficient of a 66-year term.

Table IX. tabulates for values of a proceeding by intervals of 10° from 10° to 900° t the values of $\Sigma\epsilon_t \cos at$, the summation extending over all the values of ϵ_t in Table VII.

TABLE IX.

$\frac{1}{10}\alpha$.	$\Sigma\epsilon_t \cos \alpha t$.	$\frac{1}{10}\alpha$.	$\Sigma\epsilon_t \cos \alpha t$.	$\frac{1}{10}\alpha$.	$\Sigma\epsilon_t \cos \alpha t$.	$\frac{1}{10}\alpha$.	$\Sigma\epsilon_t \cos \alpha t$.
1°	+184	24°	+308	46°	−31	68°	+29
2	+183	25	+299	47	−22	69	+32
3	+182	26	+286	48	−12	70	+35
4	+181	27	+267	49	− 2	71	+38
5	+180	28	+247	50	+ 6	72	+40
6	+179	29	+221	51	+16	73	+44
7	+182	30	+194	52	+21	74	+44
8	+182	31	+164	53	+27	75	+47
9	+185	32	+134	54	+28	76	+48
10	+189	33	+104	55	+33	77	+46
11	+196	34	+ 76	56	+32	78	+45
12	+205	35	+ 47	57	+33	79	+42
13	+217	36	+ 26	58	+30	80	+40
14	+229	37	+ 2	59	+30	81	+38
15	+244	38	− 16	60	+27	82	+37
16	+257	39	− 30	61	+24	83	+34
17	+272	40	− 40	62	+23	84	+31
18	+285	41	− 47	63	+24	85	+29
19	+297	42	− 47	64	+22	86	+30
20	+305	43	− 48	65	+24	87	+29
21	+312	44	− 44	66	+24	88	+27
22	+314	45	− 39	67	+26	89	+30
23	+315					90	+31

The maximum at $\alpha = 230° t$ is very striking. If $b \cos \beta t$ be added to each quantity ϵ_t, the increment of $\Sigma\epsilon_t \cos \alpha t$ is

$$8b[F(\alpha-\beta)+F(\alpha+\beta)]$$

when

$$F(x) = \frac{\sin \frac{16x}{20}}{16 \sin \frac{x}{20}} \cos \frac{15x}{20}$$

Table X. tabulates $10^3 . F(x)$ for each degree of $\frac{x}{10}$ from 0° to 90°.

TABLE X.

Values of $\dfrac{\sin \frac{16x}{20}}{16 \sin \frac{x}{20}} \cos \frac{15x}{20} = \mathrm{F}(x)$.

$\frac{x}{10}$°	1000F(x).	$\frac{x}{10}$°	1000F(x).	$\frac{x}{10}$°	1000F(x).	$\frac{x}{10}$°	1000F(x).
0	+1000	23	+ 22	46	+ 22	69	+21
1	+ 989	24	+ 63	47	+ 43	70	+36
2	+ 953	25	+ 98	48	+ 62	71	+50
3	+ 897	26	+126	49	+ 79	72	+62
4	+ 822	27	+145	50	+ 92	73	+72
5	+ 731	28	+155	51	+100	74	+79
6	+ 628	29	+156	52	+102	75	+86
7	+ 517	30	+148	53	+100	76	+82
8	+ 403	31	+132	54	+ 93	77	+77
9	+ 290	32	+110	55	+ 81	78	+70
10	+ 183	33	+ 84	56	+ 67	79	+60
11	+ 85	34	+ 55	57	+ 50	80	+48
12	0	35	+ 27	58	+ 32	81	+35
13	− 70	36	0	59	+ 15	82	+22
14	− 123	37	− 23	60	0	83	+10
15	− 159	38	− 41	61	− 13	84	0
16	− 177	39	− 53	62	− 22	85	− 8
17	− 179	40	− 59	63	− 27	86	−13
18	− 166	41	− 58	64	− 28	87	−14
19	− 141	42	− 50	65	− 24	88	−12
20	− 107	43	− 37	66	− 17	89	− 8
21	− 66	44	− 20	67	− 6	90	0
22	− 22	45	0	68	+ 7		

Since $\mathrm{F}(x) = 0$ when $x = 225°$, $\mathrm{F}(\alpha-\beta)+\mathrm{F}(\alpha+\beta)$ is small when β is small, if α is nearly 225°.

From this I infer that long-period terms with large coefficients (10″ to 20″) do not greatly affect the term with argument about $230° t$ and coefficient 2″ that is indicated by Table IX.

Also the maximum in Table IX. at $230° t$ cannot be due to a term of moderate period different from itself. For such a term

would produce a maximum in Table IX. opposite its own argument far more pronounced than any effect it can produce at the point $a = 230° t$.

Table IX. is clearly not suitable for further investigation. Its ninety terms are more cumbrous in use than the sixteen terms of Table VII. The method I am now about to develop is the one by which I actually found the term $2''{\cdot}3 \cos 246° t$, and the success of the method is due to the fact that it narrows the investigation from the sixteen terms of Table VII. down to four quantities, which I call x_2, x_4, x_6, x_8.

My original idea was to develop the errors in powers of the time. In the short-period terms the high powers of the time will have coefficients that rise in importance relatively to the coefficients of the long-period terms. In fact

$$\cos pt = 1 - \tfrac{1}{2}p^2t^2 + \tfrac{1}{24}p^4t^4 - \tfrac{1}{720}p^6t^6 + \tfrac{1}{40320}p^8t^8$$

and the method in its simplest form is to take advantage of the factor p^8 in the coefficient of t^8. It was convenient, however, to make one modification. If we equate the errors first to

$$a + \beta t^2 + \gamma t^4$$

and next to

$$a + \beta t^2 + \gamma t^4 + \delta t^6$$

we shall not get the same values of a, β, γ when we introduce δt^6 as when we omit it. In other words, if we form normal equations for a, β, γ, δ the cross terms do not vanish. I have remedied this inconvenience by resolving for quantities x_0, x_1, . . . x_9, instead of coefficients of the powers of t. The error for any value of t is equated to $\Sigma_r x_r t_r$ when t_r is rational integral algebraic function of t of the rth degree defined by the conditions

$$\Sigma_t t_r t_s = 0 \text{ when } r - s \text{ is even}, \quad \Sigma_t t_r^2 = 1.$$

The odd and even powers of t can be separated as before, and the quantities given in Tables IV. and V. are still those appropriate to even and odd function analysis. The summation Σ extends over the sixteen values of $10t$ from 0 to 15.

t_{2r} will consist of even powers of t only; t_{2r+1} of odd powers only. When the quantities t_r have been calculated, first in powers of t and then numerically for different values of t, we have

$$x_{2r} = \Sigma \epsilon_t . t_{2r} \text{ from Table VII.}$$

$$x_{2r+1} = \Sigma \epsilon_t' t_{2r+1} \text{ from Table VIII.}$$

and the probable errors of the quantities x are equal to those of the quantities ϵ_t.

Now, for the calculation of the quantities t_r, let S_{2m} denote Σt^{2m}, so that

$$S_0 = 16, \; S_2 = 12{\cdot}4 \text{ &c.}$$

Consider the two determinants

$$\begin{vmatrix} S_{18} & S_{16} & S_{14} & S_{12} & S_{10} & S_8 \\ S_{16} & S_{14} & S_{12} & S_{10} & S_8 & S_6 \\ S_{14} & S_{12} & S_{10} & S_8 & S_6 & S_4 \\ S_{12} & S_{10} & S_8 & S_6 & S_4 & S_2 \\ S_{10} & S_8 & S_6 & S_4 & S_2 & S_0 \end{vmatrix}$$

obtained by suppressing the first or last column in the above form. From the former, when the first column is suppressed, the coefficients of t_8 in powers of t can be calculated, and from the latter the coefficients of t_9. The formulæ are perfectly general, but I have not gone beyond t_9.

Let $D_{2m.2n.2p}$ denote a determinant of three rows and columns, three being the number of suffixes chosen for illustration, built up on S_{2m}, S_{2n}, S_{2p} as a base, and the elements standing in the line above having suffixes greater by 2, and in the line above that greater by 4, then

$$t_9 = D_{8.6.4.2}t^9 - D_{10.6.4.2}t^7 + D_{10.8.4.2}t^5 - D_{10.8.6.2}t^3 + D_{10.8.6.4}t$$

divided by $\{D_{10.8.6\,4.2}D_{8.6.4.2}\}^{\frac{1}{2}}$

and $t_8 = D_{6.4.2.0}t^8 - D_{8.4.2.0}t^6 + D_{8.6.2.0}t^4 - D_{8.6.4.0}t^2 + D_{8.6.4.2}$

divided by $\{D_{8.6.4.2.0} \cdot D_{6.4.2.0}\}^{\frac{1}{2}}$

These formulæ follow easily from the definitions of t_9 and t_8, and analogous formulæ for t_7 are obvious.

We have

$S_{18} = 2050{\cdot}579$	$S_{10} = 110{\cdot}65327$	$S_2 = 12{\cdot}4$
$S_{16} = 965{\cdot}4783$	$S_8 = 56{\cdot}66482$	$S_0 = 16$
$S_{14} = 460{\cdot}3402$	$S_6 = 30{\cdot}48292$	
$S_{12} = 223{\cdot}1603$	$S_4 = 17{\cdot}8312$	

The value of $D_{10.8.6.4.2}$ turns out to be about 10, and the value of $D_{8.6.4.2.0}$ about 100. As the values of the leading terms in the determinants as they stand are about 2×10^{10} and 4×10^9, it is clear that some modifications must be introduced before numerical calculation. I therefore divided the rows and columns of the determinants by successive powers of 2, beginning with the last column and the lowest row. I then subtracted from each row above the first the row immediately below it. These two modifications only alter the determinants and minors by some power of 2. The calculations now just fall within the compass of seven-figure logarithms when performed according to the direct rules. I give below the elements of the new determinants with their logarithms written below :—

+0·058409 , +0·04374804 , +0·027381968, +0·007241247, −0·020909214, −0·067203389
[8·7664798]

+0·04374804 , +0·027381968, +0·007241247, −0·020909214, −0·067203389, −0·16185875
[8·6409586]

+0·027381968, +0·007241247, −0·020909214, −0·067203389, −0·16185875 , −0·43555
[8·4374647]

+0·007241247, −0·020909214, −0·067203389, −0·16185875 , −0·43555 , −2·45
[7·8598133] , [8·3203377] , [8·8273912] , [9·2091362] , [9·6390380]

0·86447865 , 0·88538786 , 0·95259125 , 1·11445 , 1·55 , 4·0
[9·9367542] , [9·9471336] , [9·9789066] , [0·0470606]

I use $\Delta_{2m.2n.2p}$, with a definition similar to $D_{2m.2n.2p}$. We have

$$D_{2m.2n.2p} = 2_{m+4+n+3+p+2}\Delta_{2m.2n.2p},$$

whence

$$t_9 = \Delta_{8.6.4.2}t^9 - 2\Delta_{10.6.4.2}t^7 + 2^2\Delta_{10.8.4.2}t^5 - 2^3\Delta_{10.8.6.2}t^3 + 2^4\Delta_{10.8.6.4}t$$

divided by $\{2^{11} \cdot \Delta_{10.8.6.4.2} \cdot \Delta_{8.6.4.2}\}^{\frac{1}{2}}$

and

$$t_8 = \Delta_{6.4.2.0}t^8 - 2 \cdot \Delta_{8.4.2.0}t^6 + 2^2\Delta_{8.6.2.1}t^4 - 2^3\Delta_{8.6.4.0}t^2 + 2^4\Delta_{8.6.4.2}$$

divided by $\{2^{10} \cdot \Delta_{8.6.4.2.0} \cdot \Delta_{6.4.2.0}\}^{\frac{1}{2}}$.

As the calculation is a very long one, I give an outline of it.

$\Delta_{2.0}$ = −1·7422 + 3·7975 = 2·0553 = [0·3128752]
$\Delta_{4.0}$ = −0·6474350 + 2·7304025 = 2·0829675 = [0·3186825]
$\Delta_{6.0}$ = −0·2688136 + 2·3338486 = 2·0650350 = [0·3149275]
$\Delta_{8.0}$ = −0·0836369 + 2·1692003 = 2·0855634 = [0·3192234]
$\Delta_{4.2}$ = −0·2508811 + 0·4853988 = 0·2345176 = [9·3701755]
$\Delta_{6.2}$ = −0·1041652 + 0·4149011 = 0·3107359 = [9·4923914]
$\Delta_{8.2}$ = −0·0324093 + 0·3856307 = 0·3532214 = [9·5480470]
$\Delta_{10.2}$ = +0·0112239 + 0·3765236 = 0·3877475 = [9·5885490]
$\Delta_{6.4}$ = −0·07489483 + 0·15418524 = 0·07929041 = [8·8992207]
$\Delta_{8.4}$ = −0·02330228 + 0·14330778 = 0·12000550 = [9·0792011]
$\Delta_{10.4}$ = +0·00807001 + 0·13992342 = 0·14799343 = [9·1702424]
$\Delta_{8.6}$ = −0·01991794 + 0·05950107 = 0·03958313 = [8·5975101]
$\Delta_{10.6}$ = +0·00689795 + 0·05809589 = 0·06499384 = [8·8128722]
$\Delta_{10.8}$ = +0·00641131 + 0·01807557 = 0·02448688 = [8·3889335]

$\Delta_{4.2.0}$ = 10⁻⁷{−1381231 + 3371465 − 1021441 = 968793} = [8·9862310]
$\Delta_{6.2.0}$ = 10⁻⁷{− 429747 + 3342441 − 1353410 = 1559284} = [9·1929250]
$\Delta_{8.2.0}$ = 10⁻⁷{+ 148829 + 3375667 − 1538456 = 1986040} = [9·2979880]
$\Delta_{6.4.0}$ = 10⁻⁸{−4355321 + 13877737 − 3453495 = 6068921} = [8·7831114]
$\Delta_{8.4.0}$ = 10⁻⁸{+1508328 + 14015694 − 5226839 = 10297182} = [9·0127184]
$\Delta_{8.6.0}$ = 10⁻⁸{+1495343 + 4360749 − 1724043 = 4132049} = [8·6161654]

$\Delta_{6.4.2} = 10^{-9}\{-4903580+20882505-12833849 = 3145076\} = [7{\cdot}4976312]$
$\Delta_{8.4.2} = 10^{-9}\{+1698200+23737678-19423937 = 6011941\} = [7{\cdot}7790147]$
$\Delta_{10.4.2} = 10^{-8}\{+642155+2605795-2395403 = 852547\} = [7{\cdot}9307184]$
$\Delta_{8.6.2} = 10^{-9}\{+2250115+7385581-6406876 = 3228820\} = [7{\cdot}5090438]$
$\Delta_{10.6.2} = 10^{-8}\{+850856+810749-1051982 = 609623\} = [7{\cdot}7850614]$
$\Delta_{10.8.2} = 10^{-9}\{+9671898-2807774-3963416 = 2900708\} = [7{\cdot}4625040]$
$\Delta_{8.6.4} = 10^{-9}\{+574161+2509219-2660120 = 423260\} = [6{\cdot}6266072]$
$\Delta_{10.6.4} = 10^{-9}\{+2171128+3094426-4367807 = 897747\} = [6{\cdot}9531540]$
$\Delta_{10.8.4} = 10^{-9}\{+3285987-1071657-1645602 = 568728\} = [6{\cdot}7549046]$
$\Delta_{10.8.6} = 10^{-9}\{+1083864-470636-512001 = 101227\} = [6{\cdot}0052964]$

$\Delta_{6.4.2.0} = 10^{-9}\{+701527+3260338-4078520+509058 = 392403\} = [6{\cdot}5937323]$
$\Delta_{8.4.2.0} = 10^{-9}\{+2652746+4152654-6920056+973085 = 858429\} = [6{\cdot}9337044]$
$\Delta_{8.6.2.0} = 10^{-9}\{+4269625-1438140-2776877+522613 = 577221\} = [6{\cdot}7613422]$
$\Delta_{8.6.4.0} = 10^{-9}\{+1661790-745644-863979+68508 = 120675\} = [6{\cdot}0816173]$
$\Delta_{8.6.4.2} = 10^{-11}\{+8611840-4353394-6751208+2844450 = 351688\} = [4{\cdot}5461575]$
$\Delta_{10.6.4.2} = 10^{-10}\{+1375909-617350-1274674+603316 = 87201\} = [4{\cdot}9405215]$
$\Delta_{10.8.4.2} = 10^{-10}\{+2630106-2334442-606515+382204 = 71353\} = [4{\cdot}8534122]$
$\Delta_{10.8.6.2} = 10^{-10}\{+1412545-1669268+210047+68028 = 21352\} = [4{\cdot}3294386]$
$\Delta_{10.8.6.4} = 10^{-10}\{+1851680-2458208+411830+211658 = 16960\} = [3{\cdot}2294258]$

$\Delta_{8.6.4.2.0} = 10^{-11}\{+1716686-2350548+417980+252319-23635 = 12802\} = [3{\cdot}1072778]$
$\Delta_{10.8.6.4.2} = 10^{-11}\{+20542-38149+19538-1546-355 = 30\} = [0{\cdot}4771213]$

whence

$$t_0 = 0{\cdot}25$$
$$t_1 = 0{\cdot}2840\, t$$
$$t_2 = 0{\cdot}3488\, t^2 - 0{\cdot}2703 \quad [9{\cdot}54253]$$
$$t_3 = 0{\cdot}454\, t^3 - 0{\cdot}6535\, t \quad [9{\cdot}65750]$$
$$t_4 = 0{\cdot}5768\, t^4 - 1{\cdot}1670\, t^2 + 0{\cdot}2628 \quad [9{\cdot}76023] \quad [0{\cdot}06707]$$
$$t_5 = 0{\cdot}763\, t^5 - 2{\cdot}023\, t^3 + 1{\cdot}0322\, t \quad [9{\cdot}88267] \quad [0{\cdot}30591]$$
$$t_6 = 0{\cdot}9820\, t^6 - 3{\cdot}1612\, t^4 + 2{\cdot}4608\, t^2 - 0{\cdot}2550 \quad [9{\cdot}99213] \quad [0{\cdot}49985] \quad [0{\cdot}39107]$$
$$t_7 = 1{\cdot}322\, t^7 - 5{\cdot}053\, t^5 + 5{\cdot}427\, t^3 - 1{\cdot}4229\, t \quad [0{\cdot}12110] \quad [0{\cdot}70352] \quad [0{\cdot}73457]$$
$$t_8 = 1{\cdot}7301\, t^8 - 7{\cdot}5697\, t^6 + 10{\cdot}1800\, t^4 - 4{\cdot}2565\, t^2 + 0{\cdot}2481 \quad [0{\cdot}23808] \quad [0{\cdot}87908] \quad [1{\cdot}00775] \quad [0{\cdot}62905]$$
$$t_9 = 2{\cdot}3925\, t^9 - 11{\cdot}8644\, t^7 + 19{\cdot}4164\, t^5 - 11{\cdot}6208\, t^3 + 1{\cdot}8460\, t \quad [0{\cdot}37885] \quad [1{\cdot}07425] \quad [1{\cdot}28817] \quad [1{\cdot}06522]$$

and the numerical values are given in Table XI.

TABLE XI.

Values of t_0, t_1 . . . t_9 *in Units of* 0·001.

10′.	t_1.	t_3.	t_5.	t_7.	t_9.
1	+ 28	− 64	+101	−137	+173
2	+ 57	−127	+190	−244	+282
3	+ 85	−184	+258	−293	+284
4	+114	−232	+291	−272	+175
5	+142	−270	+287	−181	− 11
6	+170	−294	+241	− 38	−200
7	+199	−301	+157	+126	−311
8	+227	−290	+ 40	+262	−278
9	+256	−257	− 95	+323	− 93
10	+284	−199	−228	+273	+169
11	+312	−114	−328	+ 97	+354
12	+341	+ 1	−357	−166	+282
13	+369	+150	−268	−393	−113
14	+398	+332	0	−343	−514
15	+426	+554	+518	+394	+253

10′.	t_0.	t_2.	t_4.	t_6.	t_8.
0	+250	−270	+263	−255	+248
1	+250	−266	+251	−230	+206
2	+250	−256	+217	−162	+ 94
3	+250	−238	+163	− 59	− 59
4	+250	−214	+ 91	+ 62	−202
5	+250	−183	+ 7	+177	−291
6	+250	−144	− 82	+267	−289
7	+250	− 99	−171	+308	−185
8	+250	− 47	−248	+282	− 1
9	+250	+ 13	−304	+186	+201
10	+250	+ 79	−328	+ 27	+332
11	+250	+152	−306	−165	+302
12	+250	+232	−224	−334	+ 64
13	+250	+319	− 65	−384	−295
14	+250	+414	+188	−182	−452
15	+250	+515	+552	+464	+325

Table XII. now gives x_2, x_4, x_6, x_8 for the quantities ε_t and for various comparison terms. Table XIII. gives x_1, x_3, x_5, x_7, x_9 for the quantities ε'_t and for the corresponding comparison terms.

TABLE XII.

Values of x_2, x_4, x_6, x_8 *for the Moon and Comparison Terms. Unit* 0″·05.

	x_2.	x_4.	x_6.	x_8.
ϵ_t	− 43	+ 93	− 70	+ 14
1000 cos 45° t	− 789	+ 26	+ 3	− 2
1000 cos 50° t	− 953	+ 41	− 1	− 4
1000 cos 60° t	−1294	+ 78	0	− 1
1000 cos 70° t	−1647	+ 138	− 4	− 1
1000 cos 120° t	−2787	+ 853	− 86	+ 3
1000 cos 180° t	−1536	+2198	− 608	+ 73
1000 cos 200° t	− 693	+2445	− 934	+ 147
1000 cos 220° t	+ 111	+2459	−1305	+ 264
1000 cos 230° t	+ 449	+2365	−1499	+ 345
1000 cos 240° t	+ 726	+2202	−1688	+ 439
1000 cos 250° t	+ 931	+1979	−1865	+ 546
1000 cos 260° t	+1056	+1703	−2023	+ 672
1000 cos 280° t	+1060	+1020	−2238	+ 963
1000 cos 320° t	+ 305	− 404	−2112	+1613
1000 cos 330° t	+ 42	− 680	−1949	+1766
1000 cos 340° t	− 220	− 900	−1735	+1902
1000 cos 360° t	− 666	−1131	−1181	+2098
1000 cos 390° t	− 981	− 935	− 195	+2122
1000 cos 400° t	− 971	− 749	+ 128	+2043
1000 cos 410° t	− 899	− 520	+ 419	+1918
1000 cos 438° t	− 489	+ 225	+1009	+1342

TABLE XIII.

Values of x_1, x_3, x_5, x_7, x_9 *for the Moon and Comparison Terms. Unit* 0″·05.

	x_1.	x_3.	x_5.	x_7.	x_9.
ϵ'_t	− 58	+ 40	− 33	− 4	+ 18
1000 sin 45° t	+2378	− 162	+ 2	− 1	− 7
1000 sin 50° t	+2548	− 218	+ 4	− 4	− 9
1000 sin 60° t	+2806	− 360	+ 12	− 4	− 8
1000 sin 70° t	+3248	− 546	+ 24	− 4	− 13
1000 sin 120° t	+2027	−1815	+ 290	− 23	− 3
1000 sin 180° t	− 504	−2466	+1277	− 224	+ 22
1000 sin 200° t	−1005	−2121	+1687	− 393	+ 57
1000 sin 220° t	−1153	−1512	+2038	− 632	+ 96
1000 sin 230° t	−1113	−1142	+2165	− 772	+ 130
1000 sin 240° t	−1003	− 747	+2250	− 925	+ 172

	x_1.	x_3.	x_5.	x_7.	x_9.
1000 sin 250° t	− 835	− 345	+2286	−1092	+ 224
1000 sin 260° t	− 630	+ 44	+2264	−1264	+ 291
1000 sin 280° t	− 138	+ 714	+2043	−1603	+ 456
1000 sin 320° t	+ 636	+1269	+ 975	−2085	+ 947
1000 sin 330° t	+ 721	+1219	+ 626	−2122	+1092
1000 sin 340° t	+ 753	+1096	+ 270	−2118	+1245
1000 sin 360° t	+ 656	+ 688	− 404	−1966	+1542
1000 sin 390° t	+ 232	− 108	−1115	−1377	+1903
1000 sin 400° t	+ 9	− 357	−1233	−1102	+1977
1000 sin 410° t	− 171	− 569	−1278	− 804	+2017
1000 sin 438° t	− 476	− 898	−1049	+ 85	+1937

From Table XII.

	x_0.	x_4.	x_6.	x_8.
ϵ_t	− 43	+93	−70	+14
−12″·2 cos 69° t... ...	+316	−19		
Sum	+273	+74	−70	+14
+15″·0 cos 45° t	−237	+ 8		
− 1·95 cos 246 t ...	− 33	−80	+70	−20
+ 0·15 cos 390 t ...	− 3	− 2	0	+ 6
Sum	0	0	0	0

From Table XIII.

	x_1.	x_3.	x_5.	x_7.	x_9.
ϵ'_t	− 58	+40	−33	− 4	+18
+9″·5 sin 60° t	+532	−69	+ 2	− 1	− 2
Sum ...	+474	−29	−31	− 5	+16
−9″·7 sin 45° t	−464	+32			
+0·43 sin 246 t	− 8	− 4	+20	− 9	+ 2
−0·50 sin 390 t	− 2	+ 1	+11	+14	−19
Sum ...	0	0	0	0	0

At the beginning of the analysis I took means for 17 consecutive periods. The effect of this is to diminish the coefficient of any 42-year term in the ratio 1·4 to 1, and the coefficient of any 66-year term in the ratio 1·17 to 1. Multiplying by these ratios, the empirical formula that I have given at the beginning of this paper is shown to be a solution of the observed values of the quantities x.

I give a brief outline of the investigation by which I sought a solution for the observed x's. I can only add that throughout

the entire calculations I saw no alternative to my conclusions on the supposition that the x's are free from accidental error, and no reason to suppose that small errors in the x's will produce other than small variations in the solution.

If means for 17 consecutive periods be taken in Tables IV. or V. it will be seen that, when cleared of short-period terms, the errors of the Moon are reduced to such small quantities that the probable error of the quantities ϵ_t, ϵ'_t, each representing about 2000 observations, may be taken as $0''{\cdot}2$ or 4 units.

The probable error of an x is equal to that of a quantity ϵ_t or ϵ'_t.

Hence the probable error of the x's is about 4. The observed values of x_5, x_6, x_8, x_9 are certainly real and not accidental, and the existence of at least one term of moderate period is therefore demonstrated.

Again, a comparison of x_5, x_7, x_9 with the comparison terms of Table XIII. proves the existence of at least two terms of moderate period, for no single term bears sufficient resemblance to ϵ'_t.

From the values of x_6, x_8 and from Table IX. I infer that one term of moderate period has an argument approximating to $230° t$.

An attempt to work with $2''{\cdot}5 \cos 220° t$ fails, on account of the large value of x_4 left when $2''{\cdot}5 \cos 220° t$ is taken out.

A large value of x_4 implies a large coefficient in the long-period term ; this will leave x_2 large, and an enormous secular term would have to be called in to explain x_2. All this can be avoided by increasing the argument to $246° t$.

Coming now to ϵ'_t, the odd function analysis, my search proceeded thus :—I took out a small term $k \sin 246° t$, so as to reduce x_7 to zero, and I got a certain ratio between the coefficients of x_5 and x_9. I then took out from the x's for 1000 sin $246° t$ the x's for A sin αt, giving α different values, and always giving A such a value that x_7 was zero for 1000 sin $246° t$ —A sin αt, and I took different values of α, until at last the ratio $x_5 : x_9$ was the same as for $\epsilon'_t - k \sin 246° t$. I thus found the argument $390° t$, and I then imported it into the even function analysis, to reduce the value of x_8.

Meanwhile I was also making the assumption that only one long-period empirical term was to be introduced. First I tried $60° t$; *i.e.* I permitted to myself alterations of the coefficients of Newcomb's empirical term, but no alteration of its period. I was led to $9''{\cdot}2 \cos 60° t$, an alteration of $3''$ in the even part of Newcomb's term. This coefficient $9''{\cdot}2$ is determined by the value of x_2 : it implies a definite value of x_4, and everything can be adjusted for 150 years (1750 to 1901) by taking $240° t$ instead of $246° t$. The solution, however, breaks down when the numerical values are calculated for $t = -2$, -3, -4 or the years 1736, 1691, 1646. I should say that the coefficient of the sine term is found from x_3, and any outstanding part of x_1 can be ascribed to a mean motion.

Table XIV. gives an outline of the result of taking various arguments for the long-period term, and permitting no alteration of the secular term.

TABLE XIV.

rg.	Correction to Hansen-Newcomb.	Values for $t=-2$.	$t=-3$.	$t=-4$.
$5t$	$+3''\cdot5-12''\cdot2\cos 60°\,t+9''\cdot5\sin 60°\,t+7''\cdot5\cos 70°\,t-3''\cdot0\sin 70°\,t-3''\cdot9\,t$	$+5''\cdot4$	$+19''\cdot4$	$+31''\cdot7$
$0t$	$+1\cdot5$ „ „ $+9\cdot2\cos 60\,t-4\cdot5\sin 60\,t-3\cdot0\,t$	$+4\cdot6$	$+13\cdot4$	$+19\cdot3$
$0t$	$-1\cdot6$ „ „ $+12\cdot5\cos 50\,t-6\cdot7\sin 50\,t-1\cdot8\,t$	$+4\cdot3$	$+8\cdot5$	$+5\cdot9$
$5t$	$-4\cdot3$ „ „ $+15\cdot2\cos 45\,t-9\cdot0\sin 45\,t-0\cdot5\,t$	$+3\cdot6$	$+5\cdot0$	$-3\cdot2$

It will be seen that the argument $60°\,t$ leads to errors of 20″ about 1650, and that $70°\,t$ (which is approximately Hansen's argument that Professor Newcomb removed) is worse ; $50°\,t$ is better and $45°\,t$ will do.

Since Table XIV. was computed the coefficients have undergone slight adjustments. In particular $-0\cdot5\,t$ has been replaced by $-0''\cdot7\sin 45°\,t$, from which it is, of course, indistinguishable during modern times.

Finally, having at length obtained long-period terms

$$15''\cdot0\cos 45°\,t-9''\cdot7\sin 45°\,t,$$

that will satisfy the data before 1750 reasonably well, and which will satisfy with rigour the requirements of the period 1750 to 1901, it remains to point out what latitude is still permissible, or, in other words, to what extent the solution is not unique.

The above formula may be expanded in powers of t, and it will be found that the term in t^6 is insensible from 1650 onwards, and that the term in t^5 is so small that its variations as the formula is changed are insensible. We are then left with t, t^2, t^3, and t^4. Any part of the coefficient of t^2 may be ascribed to a secular term ; the ratio of the coefficients of t^2 and t^4 will then determine the period. The period being known the coefficient of t^3 determines the coefficient of the sine term, and the t term is a mean motion. In this way the statement at the beginning of the paper was deduced, showing that twice the probable correction to the secular acceleration only alters the period from 363 years to 327 years. A small error in the observed x_4 will naturally produce some effect on the period, but the main conclusion is, I think, made out, that in addition to certain terms of moderate periods the tables require a substitution of a term of longer period for that of Professor Newcomb's.

Note on a Possible Source of Error in Measures of Star Places due to Defective Centring of the Object Glass. By H. H. Turner, D.Sc., F.R.S., Savilian Professor.

1. The object of this note is to call attention to the possible existence of a kind of error which has not, so far as I know, been hitherto noticed. It was suggested by the occurrence of a considerable systematic difference, varying with the magnitude of the star, between star-places derived from photographs taken with similar instruments at Algiers and at Paris, for the parallax of *Eros*. See pp. 38–40 of the 11th Circular. I have at present no means of knowing whether the following is the true cause of these differences, but it seems to me to be at least a possible cause, and one which may produce small systematic errors in general.

2. If the two lenses of an object glass are not correctly centred, *i.e.* if their optical axes do not truly coincide, the image of a star will be a small spectrum. For simplicity suppose there are two colours, red and blue, side by side. For a faint star the red rays will not affect the plate, and the image will be formed by blue rays only. But for bright stars both blue and red will affect the plate, and the centre of the image will therefore be displaced towards the red end as compared with the image for faint stars. There will thus be an error of place, depending on stellar magnitude, which may be in any direction, according to the nature of the defect. The error is similar in some respects to that called "guiding error," but is constant in its effects so long as the object glass is not disturbed.

3. The error may exist theoretically with either visual or photographic telescopes; but there is this reason for its being more likely to be serious in the latter case: viz. that the star images are not under regular inspection, as with a visual telescope, so that an unsymmetrical distribution of colour in the image might pass unnoticed.

4. The error can be both detected and eliminated by reversing the objective with reference to the stars and comparing plates taken in the two positions. In the case of telescopes mounted in the German fashion, which are reversed on opposite sides of the pier, comparison of measures taken east and west of the pier will show the error in each coordinate. It is to be noticed that in this case the error comes (doubled) into parallax measures, but is eliminated from mean results. For telescopes mounted English fashion, such as those at Paris and at Algiers, the ordinary plates do not furnish the means of detecting or eliminating the error (except by comparison with plates free from it).

5. The telescope at Oxford is mounted German fashion. Four years ago Mr. F. A. Bellamy, as a general precaution and without reference to this particular suggestion, took the same field of stars twice on the same plate, reversing the telescope between the exposures, and also turning the plate 180° in its holder, so that

the corresponding images are juxtaposed all over the plate. We thus had a ready means of testing the existence of this error in the Oxford glass at once. The plate was partially measured by Mr. B. Gray on November 7 and 8, taking, in the first instance the three bands across the plate between the values $y=0\cdot0$ to $5\cdot0$, $y=11\cdot0$ to $15\cdot0$, $y=21\cdot0$ to $26\cdot0$ (*i.e.* two outside strips and one central), and afterwards the omitted portions, $y=5\cdot0$ to $11\cdot0$ and $y=15\cdot0$ to $21\cdot0$. A comparison of the two sets gives a general check on the results, but has no particular significance.

The differences (corrected by linear corrections of the simple form $\Delta x=by+c$, $\Delta y=-bx+f$) were grouped according to the diameters of the photographic images and gave the following mean results :—

Plate 1623. *R.A.* $21^h\ 18^m\ +30^s$.

Mean Diam. of Image.	First Set.			Second Set.			Mean.		Total Stars.
	No. of Stars.	Δx.	Δy.	No. of Stars.	Δx.	Δy.	Δx.	Δy.	
"		"	"		"	"	"	"	
30	93	−0·06	−0·09	72	+0·03	0·00	0·00	−0·03	165
45	57	+ ·21	− ·12	61	+ ·21	+·12	+·21	·00	118
60	38	+ ·15	− ·27	34	+ ·12	+·18	+·12	− ·06	72
78	17	+ ·21	− ·12	16	+ ·18	+·03	+·21	− ·03	33
99	11	− ·12	+ ·06	9	·00	+·09	−·06	+ 0·9	20
140	5	− ·54	+ ·18	5	− ·66	−·18	−·57	·00	10

6. There seems to be possibly a sensible effect of the kind under consideration in the x coordinate ; but it must be remembered that the results for a single plate may be affected by "driving-error," and a systematic effect due to the objective can only be established by measuring a number of plates.

7. Finally, it is to be remarked that any error of this kind is likely to be altered when the lenses of the objective are separated for cleaning. Now I have published two papers (see *Monthly Notices*, lxiii. p. 56 and lxiv. p. 3) on the proper motions of bright stars relatively to faint stars, deduced from a comparison of measures made on plates taken at the University Observatory about 1893 and about 1898. At the time of writing them I did not know of any source of instrumental error likely to affect the results ; but, since the lenses of the objective were separated for cleaning in 1894 July, it seems now possible that the results given in the two papers above quoted are affected to an unknown extent by an error of the kind now indicated, and the results therein given must be accepted with this reservation accordingly, until further examination of the point can be made.

University Observatory, Oxford :
1904 *November* 11.

On a Very Sensitive Method of Determining the Irregularities of a Pivot; on the Pivot Errors of the Radcliffe Transit Circle, and their Effect on the Right Ascensions of the Radcliffe Catalogue for 1890. By Arthur A. Rambaut, M.A., Sc.D., F.R.S., Radcliffe Observer.

In the *Monthly Notices*, lv. pp. 21 and 292, the late Mr. Stone drew attention to the existence of small but sensible differences between the right ascensions of stars observed with the Oxford, Cape, and Greenwich Transit Circles. In order to examine how far these differences might be due, as had been suggested, to irregularities in the pivots of the Oxford instrument, he provided himself with a piece of apparatus for testing the figure of a pivot which had been devised by Monsieur M. Hamy, of the Paris Observatory, and described by him in the *Comptes Rendus*, cxvii. No. 20, and more fully in the *Bulletin Astronomique*, xii. p. 49. The result of his investigation was published by Stone in a short paper in the *Monthly Notices*, lvi. p. 338, in

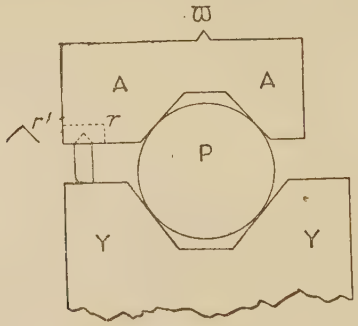

FIG. 1.

which he states that the errors are so small that they are practically insensible, and expresses his conviction that the differences in right ascension, to which attention had been called, were not due to pivot errors in the Radcliffe Transit Circle.

The apparatus employed by Stone was constructed by Messrs. Troughton and Simms and differs from Hamy's in one or two unimportant details. It was arranged as follows : a brass block A A (fig. 1) in the form of an inverted Y-bearing rides on the pivot P and is supported on a point p fixed with regard to the telescope-bearing Y. This point engages a horizontal groove r (shown in profile in r') cut in the block in a direction parallel to the meridian.

The extremity of a lever, L (fig. 2), movable about a horizontal axis, a, fixed to the pier of the telescope, bears on a point ϖ attached to the block directly above the centre of the

pivot. This lever supports a small horizontal mirror, *m*, of black glass at a convenient distance from the fulcrum *a*. The mirror stands above and very close to the upper plane face of the lens *l* of a bent collimator provided with three levelling screws which rest on the pier. At the focus of the lens *l* is placed a small total-reflexion prism, *d*, which is illuminated from the side with monochromatic light by means of a condensing lens. When this prism is adjusted so that light enters the collimator interference fringes are produced in the lamina between the mirror *m* and the lens *l* as soon as their plane faces are brought into sensible parallelism. An eye placed at E on looking at the lens *l* as reflected in the prism D will see these fringes at one side of the prism *d*.

Things being so arranged, when the telescope is turned, the block A remains immovable, or moves slightly in an up-and-down direction, oscillating about the point *p*, according as the pivot P is, or is not, a perfect surface of revolution. In the one case

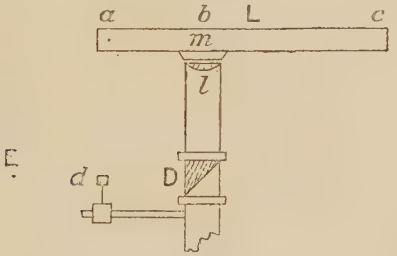

Fig. 2.

the fringes remain stationary, in the other they are displaced to a greater or less extent.

Monsieur Hamy's method is beautifully sensitive, and enables one to say almost immediately whether a pivot is sensibly true or not. As he says: "Les expériences que je vais décrire permettent de se rendre compte, en quelques instants, de l'état des tourillons d'une lunette et répondent, sur-le-champ, à la question de savoir si leur forme est assez parfaite, pour n'avoir pas à redouter d'erreur appréciable dans les mesures méridiennes."

Some time after the direction of the Radcliffe Observatory came into my hands, being anxious to make sure of this point, which seemed to me to have been rather hastily disposed of, I examined the eastern pivot with the apparatus as left by Stone, and soon detected the existence of appreciable errors which it seemed important to evaluate. The method as described by Monsieur Hamy does not, however, afford us the means of determining the amount of the errors when the pivot is found to deviate from an exactly cylindrical form, or of evaluating the effect of the errors on the time of transit of a star. For this purpose it is necessary to attack the problem in some other way.

The principal methods hitherto employed are :

(1) That of Airy, still used at the Royal Observatory, Greenwich, and fully described in the *Greenwich Observations* for 1852, Appendix I., p. {17}. It has been recently applied by Sir David Gill in a slightly modified form to the Cape Transit Circle with signal success (see *Monthly Notices*, lix. p. 125) ;
(2) That of Loewy and Périgaud, described in the Annales de l'Observatoire de Paris, *Mémoires*, xvi. ; and
(3) That of Villarceau described at length in the Annales de l'Observatoire de Paris, *Mémoires*, vii. p. 307.

All these methods entail considerable labour and long series of micrometer measures, during which there is always the danger of changes in the temperature conditions affecting the results. There was a difficulty, too, in applying any of them to the Radcliffe Transit Circle, as each required a specialised apparatus which I was unable to provide, and I was accordingly led, in the first instance, to adopt a modification of Airy's method which seemed likely to suit the special conditions afforded by the Oxford instrument. This consisted of a plane glass mirror of 8 inches aperture, silvered on the front surface, which was rigidly attached to the end of the western pivot, its plane surface being at right angles to the axis. A large collimator of $6\frac{1}{2}$ inches aperture and 7 feet focal length was directed normally upon this. The eye-end of the collimator was fitted with a micrometer carrying two screws at right angles to each other and a Bohnenberger eyepiece, by means of which the images of the spider lines in the micrometer, as formed by reflection at the mirror, could be observed, and so the angle between the normal to the mirror and the axis of the collimator could be measured, just as the nadir point of a transit circle is determined by reflection at a surface of mercury.

It is clear that if the pivots were perfect surfaces of revolution, or even if the sections of the two pivots were similar and similarly placed figures, the only movement of the reflected images, relatively to the fixed wires, as the transit circle rotated in its bearings, would be a circular one due to the tilt of the mirror, and that if any further movement were detected measures of the displacements in two directions at right angles to each other would afford materials for determining the pivot errors.

A number of measures were made with this apparatus ; but it was found that vibrations communicated through the soil and sometimes small, comparatively slow, progressive changes lasting for some minutes affected the observations to such an extent as to mask completely the extremely small displacements, due to irregularities of the pivots, of which we were in search. This method, therefore, had to be abandoned.

In order to avoid the vibrations mentioned above it seemed

that any apparatus employed for examining the pivots must be attached to the piers themselves on which the pivots rest, and ought not to stand on a separate pier, nor to be attached to the walls of the building as the collimator had been in our first experiments. This seemed to point to Villarceau's method as the most suitable ; but while considering the advisability of falling back on this a modification of Hamy's occurred to me which I decided to adopt, and which, at a very trifling cost, has enabled me to attain my object in a most satisfactory manner.

Whatever plan is adopted the essential object is to determine the movement of *any* line rigidly fixed in the material axis of the instrument with regard to *any* axes of coordinates fixed with regard to the Y's, or piers. If we could apply Hamy's method to two *points*, one at the end of each pivot, we should have the means of determining the vertical movements of the line joining them with all desirable accuracy. Or if we had two small cylinders, or pins, of perfectly circular section, one at each end of the axis and exactly in line with or parallel to each other, the same result would be attained.

The following method was accordingly adopted. A plug of brass was inserted in the opening of each pivot, fitting tightly but without strain in the aperture. The pivots are of solid steel 3 in. in diameter and perforated by holes 1·75 in. in diameter. In each plug was firmly fixed a very carefully turned pin of hardened steel of about 1 mm. diameter. The block A and pin p of Hamy's method were of course discarded, and instead a small bracket was attached to the lever L carrying at its extremity a knife-edge of hardened steel which rested on the pin.

The arrangement of the apparatus as finally employed is shown in fig. 3, Plate 2. For the purpose of this illustration the instrument was mounted on a wooden frame carrying a wooden model of the pivot, as it was found impossible to photograph the whole apparatus *in situ*.

In Monsieur Hamy's method only the vertical movement of the pivot is considered, but for evaluating the pivot errors the horizontal displacements are of equal importance. For observing these a crank-lever, of the form shown in fig. 4, was pivoted on a fixed centre vertically above the pin. The arms of this lever were perpendicular to each other, and each carried a hardened steel straight-edge, of which one, the vertical, bore against the pin, whilst the other, the horizontal, supported the knife-edge and lever L. These two straight-edges were set accurately at right angles to each other, and so that, if produced, the straight lines which they determine would intersect at the centre on which the lever turns. They were also graduated so that the knife-edge of the lever L could be set at exactly the same distance from the centre as the pin. Any small horizontal displacement of the latter was thus converted into an equal vertical movement of the lever L, and could be observed in the same way as the vertical movements.

As it was essential to keep the lever L sensibly horizontal in these experiments a shorter knife edge was necessary when this crank-lever was in action. For convenience the two knife-edges were fixed on opposite sides of the main lever, so that in passing from

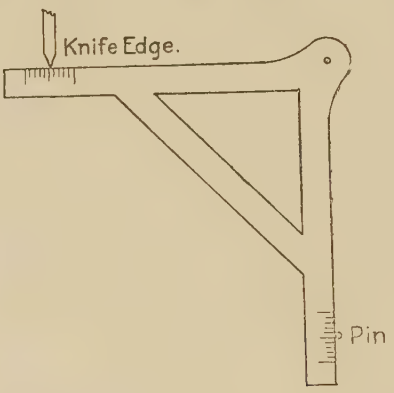

Fig. 4.

observations of the horizontal to observations of the vertical movements it was merely necessary to reverse the latter on its bearings.

Fig. 5, Plate 2, illustrates the apparatus as arranged for observations of the horizontal displacements.

The necessity of substituting pins for points at the extremities of the pivots lays this method open to two possible sources of error, against which it is necessary to guard. These are (1) errors in the figure of the pins themselves, and (2) a want of absolute accuracy in the parallelism or alignment of the two pins.

Let us consider these two sources of error separately.

(1) *Errors in the figure of the pins.*—As any deviation from the exactly circular form in that section of a pin on which the knife-edge bears will affect with its full force all observations made with it, the pins must be selected with all possible care. The figure of each pin must be critically examined before using it, and, fortunately for this purpose, we have in an application of Monsieur Hamy's method a test which leaves nothing to be desired.

The arrangements made for examining the pins were as follows :—A small lathe-bed (fig. 6, Plate 3) was placed on an isolated stone pier. To the hand-rest of this lathe was firmly attached a brass block carrying the opposing screws which form the fulcrum *a* of the lever L. On the slide-rest of the lathe was mounted a frame known as a "watchmaker's turn." One of the centres of this was of the ordinary form, and consisted of a small drilled hole, *h*, (fig. 7). The other was of brass, and was made in the form of a very small Y-bearing in which the pin itself turned ; whilst the point of the pin bore against a small flat plate of steel, *s*, let into the brass. The pin was thus supported at one

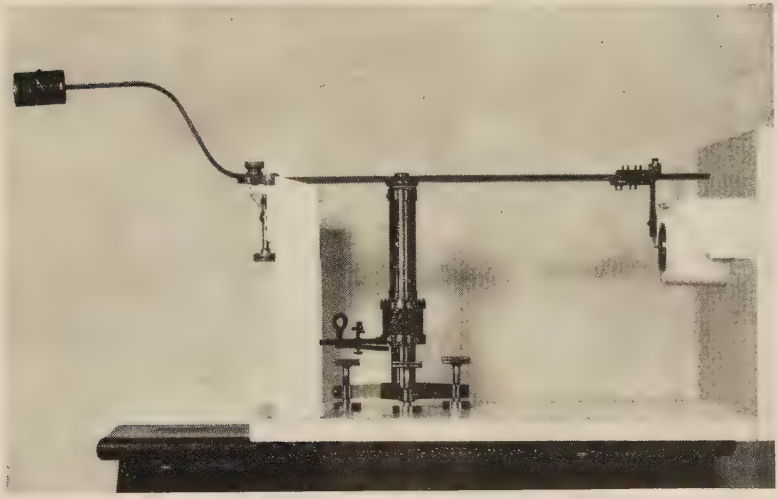

FIG. 3.—APPARATUS AS ARRANGED FOR THE VERTICAL CO-ORDINATE.

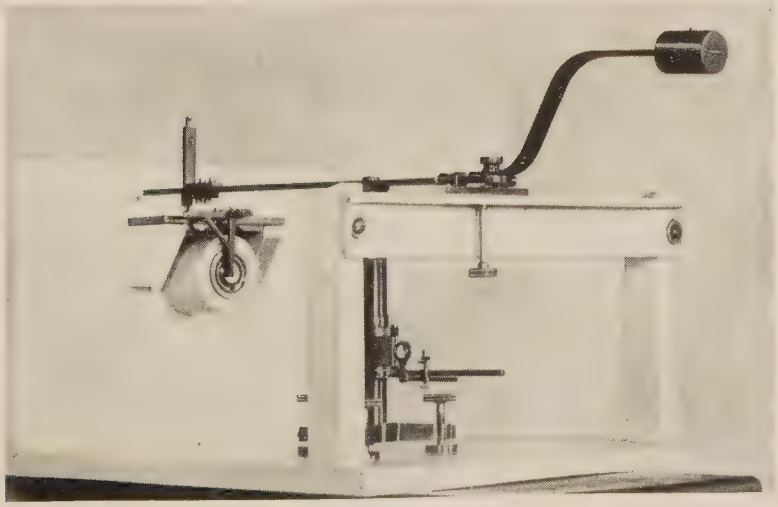

FIG. 5.—APPARATUS AS ARRANGED FOR THE HORIZONTAL CO-ORDINATE.

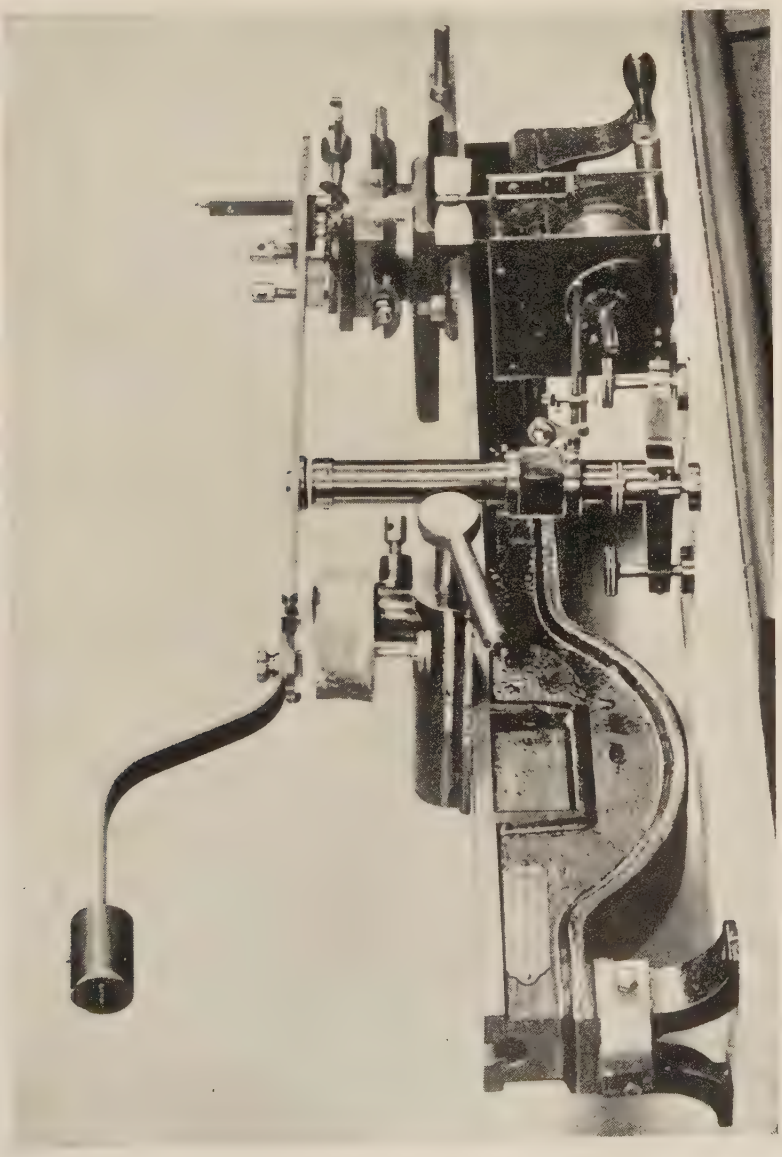

FIG. 6.—ARRANGEMENT FOR EXAMINATION OF THE PINS.

end on the dead centre on which it had been turned, and at the other it was wholly supported by the bearing at *y*. The knife-edge attached to the lever L was then allowed to bear on the pin.

For the purpose of rotating the pin smoothly a grooved wheel of boxwood, W, was screwed on to the pin and a silk thread passed over the wheel one end of which supported a small weight of 40 grammes, whilst the other was attached to a small drum driven by a simple clock-train which wound up the weight, drawing the thread over the wheel, thus causing the pin to rotate under the knife-edge. At first considerable difficulty was experienced in obtaining sufficiently smooth motion, but this was eventually traced to friction at the knife-edge, and when the weight there was relieved by a counterpoise this difficulty entirely disappeared.

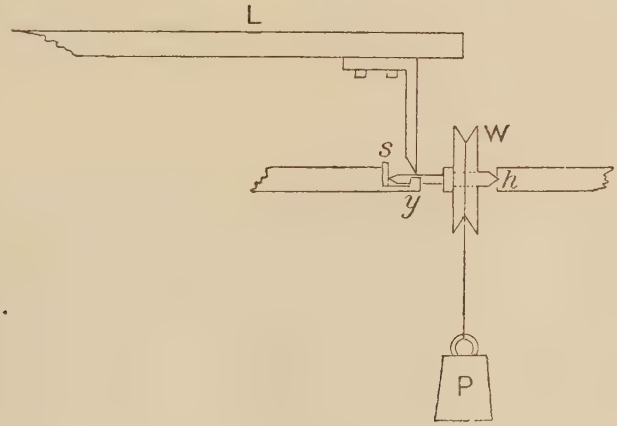

Fig. 7.

With this apparatus we were able to detect inequalities in the form of the pin which did not exceed a small fraction of a wave-length of sodium light. In fact, the test was all that could be desired. It was rapid, the complete examination of a pin occupying at most five minutes, and decisive, the same displacements of the lever reproducing themselves with monotonous regularity at each successive rotation of the pin.

A number of pins were made for me by a skilful Oxford watchmaker, Mr. H. Minn, who took every precaution to obtain a pin as nearly perfect as possible. After several attempts, however, in view of the apparent impossibility of obtaining pins absolutely free from irregularity when examined by such an exceedingly delicate test, I decided to be satisfied with two which were so nearly perfect that the total range of motion of the mirror, as either of them turned under the knife-edge, was contained within a single wave-length of sodium light. All the observations on which the results given in this paper depend were made with these two pins.

(2) *Effect of an inclination of the pin to the axis of the telescope.*—Since the movements of *any* two points, one in each pivot, will afford us the material for determining the pivot errors (see p. 316 of the memoir by Villarceau referred to above) it is clear that a mere eccentricity in the position of the pin can

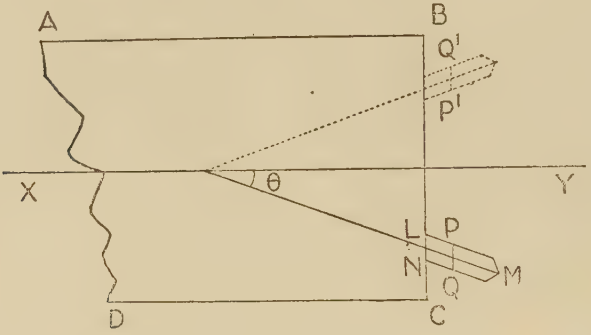

Fig. 8.

have no effect on the result. Let us next consider the effect of an inclination of the axis of the pin to the axis of rotation of the telescope. Let ABCD (fig. 8) represent one of the pivots, XY the line joining the centres of the two pivots, LMN a pin whose axis makes an angle θ with XY. Then if the knife-edge initially bears on the pin at P, it will, as the whole system rotates around XY, trace on the pin the section PQ, bearing at Q when the telescope has turned through 180° and the pin taken up the position P′Q′. This section PQ will, of course, be an ellipse, and

Fig. 9.

the movement of the lever will be exactly the same as if a cylindrical pin of elliptical section equal to PQ, but with axis parallel to XY, were substituted for the actual pin.

Now let BC, fig. 9, represent the end of the pivot, PRQ the elliptic section of the pin, and E the centre of this ellipse. The

greatest errors affecting our results occur at the points P and Q, and in linear measure they amount to EP—ER. But EP =ER sec θ; therefore the greatest error introduced by the inclination of the pin at an angle θ to the axis is

$$\text{ER}\ (\sec\theta - 1).$$

But since θ will always be a small angle we may write this

$$\tfrac{1}{2}\,\text{ER}\,.\,\theta^2.$$

We can easily ensure that θ shall not exceed 0·01. Hence we find

$$\tfrac{1}{2}\,\text{ER}\,.\,\theta^2 = 0{\cdot}00005\,.\,\text{ER}.$$

Since the pins employed are about a millimetre in diameter, or ER=0·5 mm., we find that the greatest possible error in the height of the knife-edge arising from the inclination of the pin cannot exceed

$$0{\cdot}000025 \text{ mm},$$

a quantity which we can afford to neglect.

The observations of pivot errors required the co-operation of two persons. One sat opposite the collimator and counted the fringes aloud, his whole attention being required for this. The other stood at the setting circle and rotated the telescope. The observations were made by Messrs. McClellan, Robinson, Wickham, and myself in about the proportion of 8, 1, 8 and 9 respectively. Starting from the setting 180° N.P.D., the innermost fringe was numbered 100 (to avoid negative numbers). The first fresh fringe appearing inside this was reckoned 101, the next 102, and so on. On the other hand, if the initially innermost fringe disappeared, the fringe taking its place was numbered 99. If it in turn vanished, the next outside it on assuming the position of innermost fringe was numbered 98, and so on. No attempt was made to subdivide the interval between two fringes. A change of one fringe represents a movement of half a wave-length of sodium light in the height of the black glass mirror, which corresponds to rather less than $0^s{\cdot}01$ in the time of transit of a star at the equator. At every fifth degree of N.P.D. the telescope was brought to rest, and the number of the fringe then innermost was entered. In this way, when all was going well, the displacements of one pivot in either coordinate could be investigated for a complete revolution of the telescope within a quarter of an hour without difficulty.

It should be stated that the light was obtained from an ordinary incandescent gas burner from which the mantle had been removed, and which was provided with a small platinum cup containing borax. In this way a very steady sodium flame was easily and cheaply obtained. In order to prevent any heating effects, a large sheet of glass was inserted between

the lamp and the apparatus, and for a similar reason the instrument was shielded from the heat of the observer's body by a large sheet of cardboard through a hole in which the observations were made.

The telescope was turned in both directions, but the change from one direction to another was not made until a series of observations was completed, and both coordinates of one pivot were always observed before changing the apparatus over to the other. That is to say the observations always took place in some such order as the following :—

Direct ...	East	Horizontal		Retrograde	West	Vertical	
		Vertical				Horizontal	
	West	Vertical	or		East	Horizontal	
		Horizontal				Vertical	
Retrograde	West	Horizontal		Direct ...	East	Vertical	
		Vertical				Horizontal	
	East	Vertical			West	Horizontal	
		Horizontal				Vertical	

In this way a complete direct or complete retrograde series could be obtained within a couple of hours under practically identical conditions as to temperature. Notwithstanding these precautions, however, it was sometimes found that, when the telescope had been turned right round through 360° the count of fringes did not return exactly to 100. This seemed to be due either to temperature changes, or to some slight settling down of the instrument, most probably the latter, as it did not usually occur in the first set of observations in the morning when they were made without disturbing the instrument from the position it had occupied during the night. A somewhat similar effect is referred to in Villarceau's memoir where he says (p. 323) : "Lors des observations de la fin de mars et du commencement d'avril 1860 les cinq valeurs des coordonnées obtenues pour la hauteur zéro ont présenté une marche évidemment progressive ; on en a alors déduit de petites corrections qui ont été appliquées aux mesures, afin de les ramener à la simultanéité."

To whatever cause this discrepancy may have been due (and I am inclined to attribute it chiefly to a sagging of the lever) the change seems to have taken place very nearly proportionately to the time, as is shown by the remarkably close agreement between the direct and retrograde results when corrections are applied on this hypothesis, and in the mean the effect will practically disappear.

On the other hand, the effect of personality on the part of the observer in making these observations is practically zero. There can be no doubt about the appearance or disappearance of a fresh fringe, and it was only when vibrations due to wind or heavy traffic made the fringes tremulous that the least

uncertainty arose. When this occurred the observations were rejected.

In order to test this point Mr. McClellan and I made two independent series of observations of all four coordinates. The observations were arranged as follows. First, I observed at the collimator the vertical coordinates of the east pivot, the telescope being moved first in one direction and then in the other, whilst Mr. McClellan rotated the telescope and entered the observations at every fifth degree. In Table I., column 2, are given the sums of the numbers observed by me in the two directions of rotation for every tenth degree, the intermediate observations being omitted for economy of space. Then we changed places, Mr. McClellan observing at the collimator whilst I rotated the telescope. The sums of the direct and retrograde observations made by him are entered in a similar manner in the third column of the table. The horizontal coordinate was then observed by each of us independently, the results being entered in the fourth and fifth columns of the table ; and similarly for the horizontal and vertical coordinates of the western pivot. These observations were made with two untested pins, of which one was afterwards found to be very irregular, and were subsequently rejected in deducing the errors of the pivots ; but their value for testing the effect of personality is not depreciated by defectiveness of the pins. At the foot of the table are given the total range and the greatest differences between the figures in the corresponding series ; and, bearing in mind that a unit in these sums corresponds to a movement of the black glass mirror through a quarter of a wave-length of sodium light, I think it will be admitted that the agreement of the two series is very remarkable.

TABLE I.

Z.D.	East				West			
	Vertical.		Horizontal.		Horizontal.		Vertical.	
	A.A.R.	C.	A.A.R.	C.	A.A.R.	C.	A.A.R.	C.
°								
0	200	200	200	200	200	200	200	200
10	182	182	201	202	185	186	200	201
20	173	174	197	198	170	171	206	207
30	171	172	191	191	158	159	210	211
40	177	178	181	182	146	147	214	215
50	185	187	171	172	137	137	218	219
60	185	186	163	162	132	133	218	219
70	186	188	154	154	130	131	215	217
80	184	185	147	146	128	128	191	192
90	166	167	129	128	121	121	163	166
100	153	154	123	123	110	110	138	140
110	149	149	123	123	102	103	120	123
120	153	155	123	123	98	99	109	112

Z.D.	East				West			
	Vertical.		Horizontal.		Horizontal.		Vertical.	
	A.A.R.	C.	A.A.R.	C.	A.A.R.	C.	A.A.R.	C.
130°	168	171	126	124	97	98	101	103
140	188	190	128	127	96	97	95	98
150	203	206	124	123	96	97	93	96
160	221	224	118	116	100	101	93	95
170	236	238	112	111	117	117	85	88
180	243	247	108	106	135	137	75	78
190	241	242	107	105	151	153	58	61
200	240	240	106	104	163	165	48	51
210	244	244	108	106	176	178	46	49
220	254	256	112	109	189	191	50	53
230	268	269	117	116	201	204	57	61
240	276	277	129	126	210	212	65	67
250	284	285	145	143	216	218	74	77
260	289	290	160	157	220	222	89	92
270	288	289	170	169	225	228	104	107
280	279	280	174	174	235	237	113	116
290	269	270	178	178	245	246	127	129
300	264	266	182	180	250	252	143	145
310	263	263	185	183	250	251	161	162
320	262	262	188	187	247	249	180	180
330	252	252	190	190	242	243	191	192
340	240	240	187	186	236	238	201	202
350	225	225	188	187	218	219	201	201
360	200	200	200	200	200	200	200	200
Range	140	141	95	98	154	155	172	170
Greatest Difference	−4		+3		−3		−4	

We have seen that the effect of minute irregularities of the pins on the position of the mirror *m* does not exceed a single wave-length of sodium light. Still further, however, to reduce the possible effects of such inequalities, between each complete set of measures and the next the brass plugs carrying the pins were rotated in the pivots through a right angle. This rotation had the effect of throwing the minute errors of the pins on different parts of the pivots, and so tended to eliminate their influence from the final mean. It also introduced a new eccentricity in the position of the pin, so that the actually observed displacements were in any two cases of a totally different character. In fig. 10, Plate 4, are exhibited the horizontal displacements observed in two series of observations made on July 7

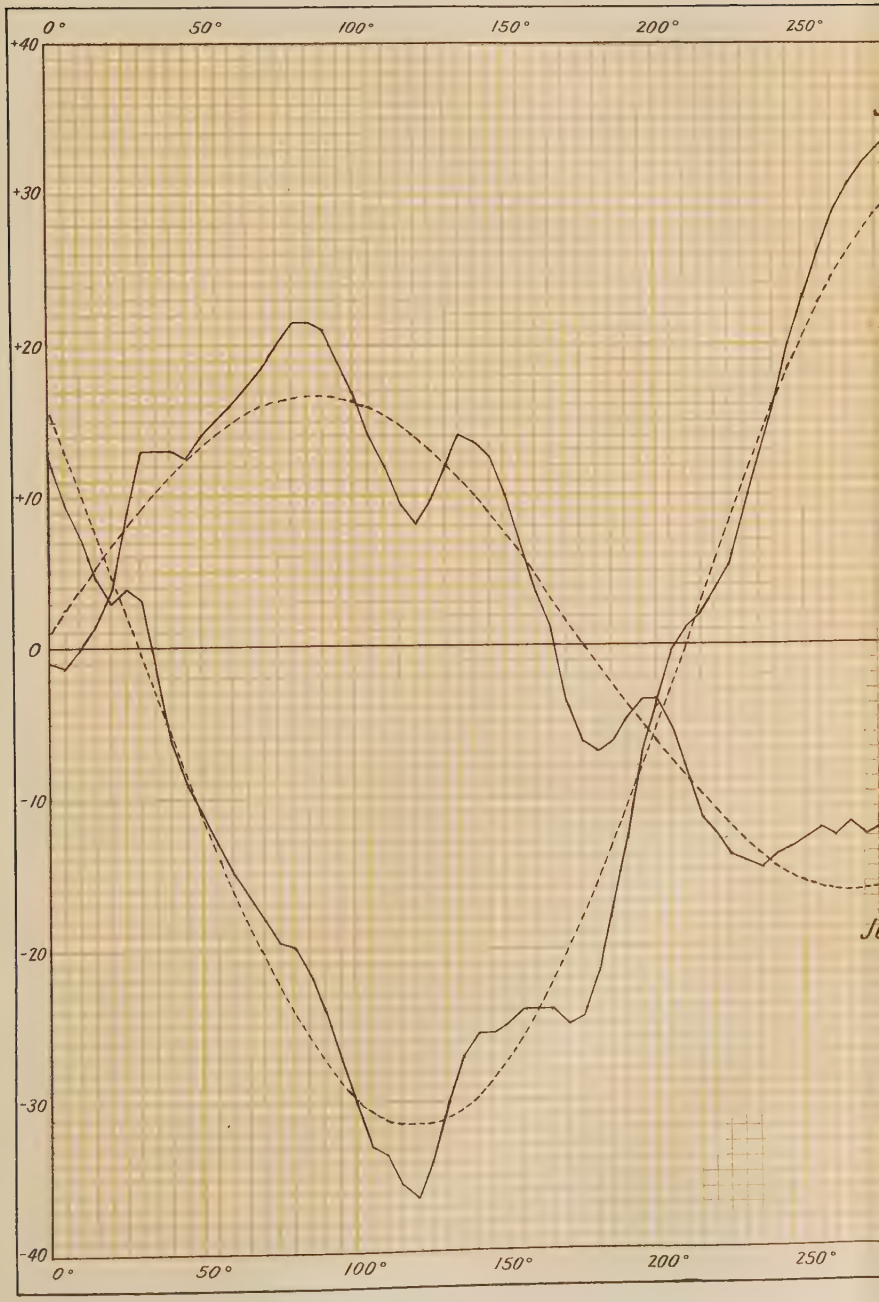

FIG. 10.—Observations of horizontal displacements, July 7 and 1

and July 12 respectively. In the figure the abscissæ represent the settings of the telescope in N.P.D., whilst the ordinates are the angular displacements of the axis of the telescope in a horizontal direction. In this diagram the side of one of the smaller squares represents a movement of $0''{\cdot}13$. At first sight it might perhaps appear that these two traces are entirely different, but it must be remembered that each is a combination of the horizontal component of a circular movement due to the eccentricity of the pin and of the superimposed effect due to the irregularities of the pivots. If therefore we denote the horizontal component of the circular movement by x, the N.P.D. by Δ, and the pivot error by δx, each observation affords us an equation of the form

$$x+\delta x = a+b\cos(\Delta+\mathrm{C}).$$

If now we determine the constants a, b, and C so as to satisfy the condition that $\Sigma(\delta x^2)$ shall be a minimum—*i.e.* if we attribute as much as possible of the observed displacements to the eccentric movement—it is easy to show, N being the number of observations made at equidistant settings of the circle, that

$$\tan \mathrm{C} = -\frac{\Sigma(x\sin\Delta)}{\Sigma(x\cos\Delta)}$$

$$a = \frac{1}{\mathrm{N}}\Sigma(x)$$

and

$$b = \frac{2}{\mathrm{N}}\cdot\frac{\Sigma(x\cos\Delta)}{\cos\mathrm{C}} = -\frac{2}{\mathrm{N}}\cdot\frac{\Sigma(x\sin\Delta)}{\sin\mathrm{C}}$$

In this way the dotted curves have been obtained, and the discrepancies between these and the observed results must be attributed to pivot errors. A remarkable agreement between the two apparently dissimilar series of observations will now become evident, and cannot fail to impress one with confidence in the accuracy of this method of determining these small and elusive irregularities.

The above comparison is given with the object of illustrating the character of the agreement between different series of observations; but in what follows, instead of keeping the horizontal and vertical components separate and calculating the effects of the pivot errors in azimuth and altitude, as is sometimes done, I have preferred to follow Villarceau in combining these into one single effect on the collimation, in virtue of which the latter can no longer be considered constant, but is affected with a variable term depending on the N.P.D. reading of the circle.

Referring to Villarceau's memoir, we find the equations for deducing the pivot errors as follows. The measured coordinates to any arbitrary origin of the centre of the pin on the eastern pivot being denoted by ξ and η, and those of the western pin by

ξ' and η' (ξ and ξ' being measured positively towards the south, η and η' positively towards the zenith), we take

$$x = \xi' - \xi \text{ and } y = \eta' - \eta.$$

Then, the observations being made at N different settings which divide exactly the circumference of the circle, we determine p and q from the equations

$$p = \frac{1}{N} \Sigma(x) \text{ and } q = \frac{1}{N} \Sigma(y).$$

If ζ denote the zenith distance, which is measured positively towards the south and continuously through 360°, and if R denote the length of the axis of the telescope between the measured points expressed in the same units as ξ, η, &c., we have N equations of the form

$$P = \{(x-p) \sin \zeta + (y-q) \cos \zeta\} / R \sin 1''.$$

We have next to take the mean, P_m, of all the separate values of P. Thus

$$P_m = \frac{1}{N} \cdot \Sigma P ;$$

and finally we have N equations of the form

$$\delta c = P - P_m$$

from which the separate values of δc are deduced, *i.e.* the variable part of the collimation depending on pivot errors.

The observations being expressed in "fringes," it is convenient to retain this unit throughout the computations, and to convert only the final results into angular measure. This is equivalent to neglecting the denominator, $R \sin 1''$, in the expression for P and calculating P, P_m, and δc in "fringes." To find the angular displacement of the axis corresponding to one of these units, we remark that one fringe takes the place of another when the distance between the upper surface of the collimator lens and the surface of the black glass mirror varies by half a wave-length, or when the difference in the lengths of the paths of the two interfering beams changes by a whole wave-length.

If, in fig. 2, a is the fulcrum of the lever, b the centre of the black glass mirror, and c the point on the lever directly above the knife-edge, the movement of b for a change of one fringe is half a wave-length ($\frac{1}{2}\lambda$); hence that of the knife-edge is $\frac{1}{2}\frac{ac}{ab} \cdot \lambda$. Hence the factor required for reducing "fringes" to seconds of arc is

$$\mu = \frac{1}{2}\frac{ac}{ab} \cdot \frac{\lambda}{R \sin 1''}.$$

From measures made on May 13 it was found that

$$R = 1382{\cdot}7^{mm}\ ;\ ac = 381{\cdot}7^{mm}\ ;\ \text{and}\ ab = 128{\cdot}8^{mm}\ ;$$

and, since λ may be taken as equal to $0{\cdot}0005893^{mm}$, we find that

$$\mu = 0''{\cdot}1302.$$

With this factor the separate results and the mean values for δc corresponding to every fifth degree in the pointing of the telescope have been reduced, as given in Table II. The last column of this table accordingly exhibits the final results of this investigation with regard to the pivot errors of the Radcliffe Transit Circle.

TABLE II.

Values of δc.

N.P.D.	July 2 and 4.	July 6.	July 7.	July 12.	Mean.
°	″	″	″	″	″
0	−0·23	−0·16	−0·05	+0·24	−0·049
5	−0·29	−0·06	−0·13	+0·25	−0·058
10	−0·20	+0·03	0·00	+0·29	+0·029
15	−0·12	+0·06	+0·04	+0·31	+0·073
20	+0·03	+0·12	+0·19	+0·29	+0·156
25	+0·38	+0·40	+0·61	+0·66	+0·513
30	+0·66	+0·70	+0·95	+0·88	+0·796
35	+0·41	+0·31	+0·60	+0·47	+0·447
40	+0·13	−0·05	+0·25	+0·06	+0·097
45	−0·08	−0·31	+0·07	−0·26	−0·143
50	−0·05	−0·44	−0·15	−0·48	−0·281
55	−0·09	−0·61	−0·35	−0·69	−0·435
60	−0·27	−0·75	−0·45	−0·92	−0·596
65	−0·26	−0·71	−0·43	−0·99	−0·598
70	−0·32	−0·72	−0·40	−1·05	−0·622
75	−0·26	−0·64	−0·42	−0·95	−0·567
80	−0·14	−0·38	−0·25	−0·94	−0·429
85	−0·14	−0·35	−0·16	−0·75	−0·351
90	−0·19	−0·21	−0·17	−0·76	−0·333
95	−0·29	−0·23	−0·19	−0·69	−0·349
100	−0·14	−0·06	−0·01	−0·59	−0·200
105	+0·05	+0·23	+0·18	−0·31	+0·037
110	+0·19	+0·43	+0·16	−0·02	+0·190
115	+0·48	+0·70	+0·49	+0·45	+0·530
120	+0·57	+1·00	+0·69	+0·69	+0·738

N.P.D.	July 2 and 4.	July 6.	July 7.	July 12.	Mean.
°	″	″	″	″	″
125	+0·34	+0·44	+0·34	+0·38	+0·374
130	−0·19	+0·07	−0·15	−0·06	−0·083
135	−0·60	−0·41	−0·46	−0·44	−0·478
140	−0·61	−0·35	−0·49	−0·45	−0·474
145	−0·33	−0·35	−0·27	−0·42	−0·344
150	−0·21	−0·14	−0·06	−0·21	−0·158
155	+0·09	+0·07	+0·12	+0·12	+0·101
160	+0·27	+0·15	+0·40	+0·44	+0·314
165	+0·39	+0·28	+0·60	+0·55	+0·455
170	+0·57	+0·32	+0·78	+0·72	+0·596
175	+0·66	+0·29	+0·82	+0·61	+0·593
180	+0·54	+0·29	+0·73	+0·58	+0·535
185	+0·35	+0·21	+0·55	+0·43	+0·385
190	+0·25	+0·09	+0·51	+0·36	+0·304
195	+0·23	+0·01	+0·38	+0·42	+0·257
200	+0·13	−0·07	+0·29	+0·21	+0·141
205	+0·07	−0·10	+0·12	+0·25	+0·088
210	+0·07	−0·16	+0·03	+0·22	+0·041
215	−0·07	−0·19	−0·19	+0·14	−0·078
220	−0·19	−0·30	−0·48	−0·13	−0·276
225	−0·30	−0·41	−0·82	−0·28	−0·452
230	−0·34	−0·27	−0·64	−0·18	−0·357
235	−0·13	−0·10	−0·61	+0·05	−0·197
240	−0·02	+0·16	−0·40	+0·21	−0·013
245	+0·21	+0·36	−0·19	+0·34	+0·180
250	+0·35	+0·57	−0·01	+0·38	+0·322
255	+0·51	+0·59	+0·08	+0·47	+0·412
260	+0·42	+0·61	+0·24	+0·42	+0·422
265	+0·39	+0·55	+0·29	+0·55	+0·445
270	+0·36	+0·53	+0·33	+0·34	+0·390
275	+0·47	+0·57	+0·27	+0·29	+0·399
280	+0·28	+0·38	+0·29	+0·12	+0·266
285	+0·23	+0·23	+0·10	+0·04	+0·151
290	+0·07	+0·18	−0·09	−0·14	+0·005
295	−0·10	+0·12	−0·14	−0·18	−0·076
300	−0·03	+0·02	−0·05	−0·15	−0·054
305	−0·06	+0·02	−0·07	−0·03	−0·036
310	−0·13	−0·04	−0·08	−0·17	−0·104
315	−0·27	−0·10	−0·19	−0·12	−0·172

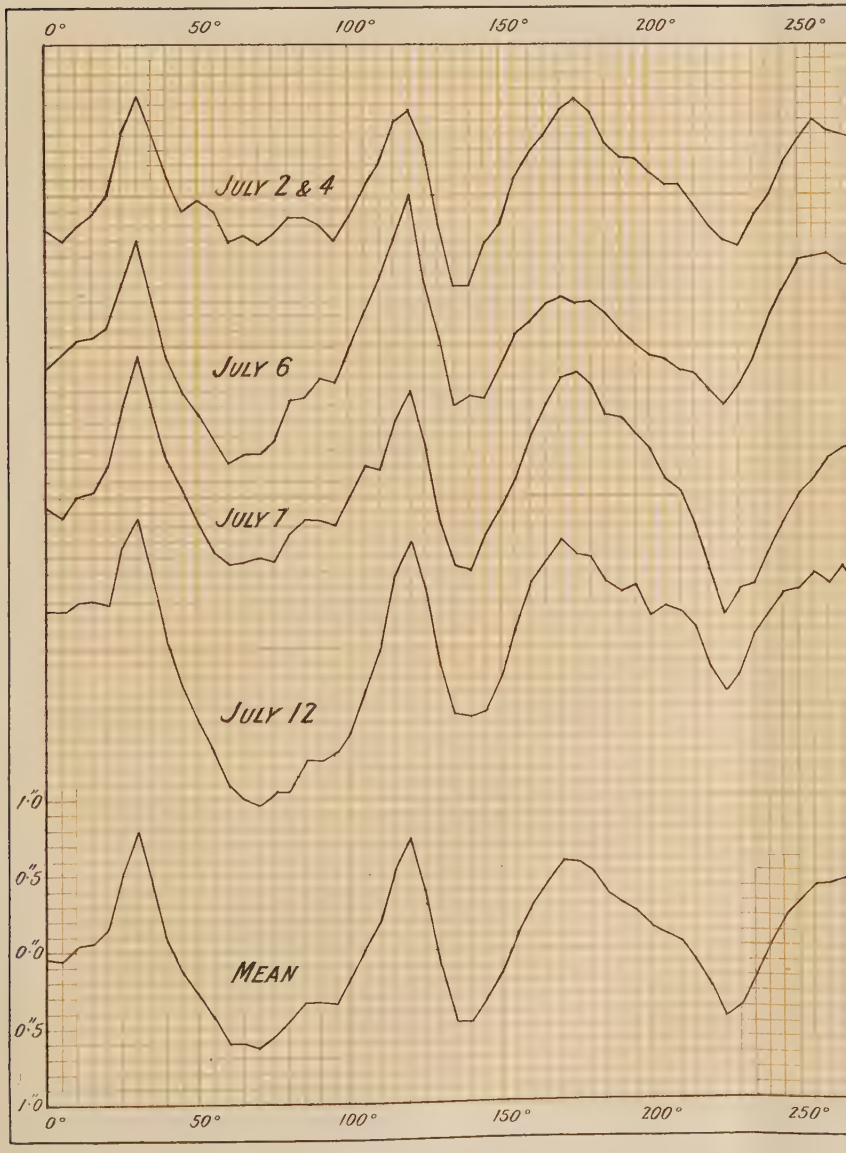

FIG. 11.—Pivot Errors of Radcliffe Transit Circle.

N.P.D.	July 2 and 4.	July 6.	July 7.	July 12.	Mean.
°	″	″	″	″	″
320	−0·29	−0·31	−0·18	−0·12	−0·221
325	−0·41	−0·23	−0·32	−0·13	−0·271
330	−0·53	−0·42	−0·46	−0·16	−0·393
335	−0·55	−0·39	−0·49	−0·23	−0·414
340	−0·52	−0·39	−0·49	−0·19	−0·398
345	−0·44	−0·40	−0·45	−0·17	−0·366
350	−0·36	−0·22	−0·24	−0·01	−0·210
355	−0·30	−0·15	−0·09	+0·09	−0·112
360	−0·23	−0·16	−0·05	+0·24	−0·049

In fig. 11, Plate 5, the separate results as given in this table have been plotted down, and the points of each series so obtained have been joined by straight lines. The means contained in the last column of the table have been treated in a similar manner, from which we obtain the fifth and lowest trace.

This diagram shows at a glance that, while there are differences between one set of observations and another, as is only to be expected when the minuteness of these quantities is borne in mind, yet the main features are repeated in a remarkable way in all four series of observations. In order, however, to form an estimate of the precision attaching to these results, I have taken the differences between each of the four individual results in Table II., and the mean, for each pointing of the telescope, and from the differences so obtained the probable error has been calculated in the usual way. I thus find that the probable error of a single determination is

$$\pm 0''\cdot 109,$$

and therefore the probable error of the means in the last column is

$$\pm 0''\cdot 054.$$

We have next to consider how these errors will affect the right ascension of a star as already computed from observations made with this instrument on the hypothesis of cylindrical pivots. For this purpose let us take

α, δ, and ζ to denote the R.A., Decl., and zenith distance of a star respectively ;

t to denote the clock time of its transit across the middle wire ;

dt to denote the correction to the clock ;

κ to denote the constant of diurnal aberration $(0''\cdot 309 \cos \phi)$;

a to denote the azimuth ;

b „ „ level ;

and c „ „ collimation.

Then equations (9) and (22) of Villarceau's memoir become

$$\alpha = t + dt + \frac{1}{15 \cos \delta}\{\psi - \kappa\}$$

and

$$\psi = a \sin \zeta + b \cos \zeta + c + \delta c.$$

Since the head of the micrometer screw of the Radcliffe Transit Circle is on the west of the tube, not on the east, as in the case imagined by Villarceau, it will be necessary, as he points out, to change the sign of k (the value of a revolution of the screw) in his subsequent formulæ. Hence, adopting his notation, according to which

l denotes the reading for coincidence of the middle wire with its reflexion at the nadir ;

m denotes the reading for coincidence of the middle wire with the wire of S. collimator ;

n denotes the reading for coincidence of the middle wire with the wire of N. collimator ;

and ν denotes the reading at which the reticule is set for observations of stars,

but putting

δc_1 for the value of δc when the telescope is pointing on S. collimator (*i.e.* $\zeta =$ *one* right angle) ;

δc_2 for the value of δc when the telescope is pointing on the nadir (*i.e.* $\zeta =$ *two* right angles) ;

and δc_3 for the value of δc when the telescope is pointing on N. collimator (*i.e.* $\zeta =$ *three* right angles),

then Villarceau's equations (23) and (26) become respectively

$$b = k(l - \nu) + c + \delta c_2$$

and

$$c = k\left(\nu - \frac{m+n}{2}\right) - \tfrac{1}{2}(\delta c_1 + \delta c_3).$$

Now if we take $\bar{c}$ as the value of the collimation deduced in the ordinary way, without regard to pivot errors, we have simply

$$\bar{c} = k\left(\nu - \frac{m+n}{2}\right) \text{ and } \therefore\ c = \bar{c} - \tfrac{1}{2}(\delta c_1 + \delta c_3).$$

Also since $k\nu = \bar{c} + k\,\dfrac{m+n}{2}$

$$\therefore\quad b = k\left(l - \frac{m+n}{2}\right) + c - \bar{c} + \delta c_2$$

$$= k\left(l - \frac{m+n}{2}\right) + \delta c_2 - \tfrac{1}{2}(\delta c_1 + \delta c_3).$$

But if the pivot errors are neglected, $k\left(l-\frac{m+n}{2}\right)$ is precisely the value we should find for the level constant. Hence, if this be denoted by $\bar{b}$ we have

$$\left.\begin{aligned} b &= \bar{b}+\delta c_2-\tfrac{1}{2}(\delta c_1+\delta c_3) = \bar{b}+\Delta b \\ \text{and} \quad c &= \bar{c}-\tfrac{1}{2}(\delta c_1+\delta c_3) \quad\quad = \bar{c}+\Delta c \end{aligned}\right\} \quad \ldots \quad \ldots \quad (1)$$

in which Δb and Δc are *constant* quantities depending on the pivot errors, given by the equations

$$\left.\begin{aligned} \Delta c &= -\tfrac{1}{2}(\delta c_1+\delta c_3) \\ \text{and} \quad \Delta b &= \Delta c+\delta c_2 \end{aligned}\right\} \quad \ldots \quad \ldots \quad \ldots \quad (2)$$

We have next to determine the correction to the azimuth constant as obtained in the usual way by observations of stars, and the error committed in deducing the clock correction by the neglect of the pivot errors.

If for simplicity we denote the azimuth, level, and collimation factors by A, B, and C; that is to say, if we take

$$\mathrm{A} = \tfrac{1}{15}\frac{\sin\zeta}{\cos\delta}\,;\ \mathrm{B} = \tfrac{1}{15}\frac{\cos\zeta}{\cos\delta}\,;\ \text{and}\ \mathrm{C} = \tfrac{1}{15}\frac{1}{\cos\delta}$$

then Villarceau's equations (9) and (22) give us for any star

$$a = t+dt+\{a\,.\,\mathrm{A}+b\,.\,\mathrm{B}+(c+\delta c-\kappa)\,.\,\mathrm{C}\}$$

substituting from equations (1) above

$$a = \{t+\bar{b}\,.\,\mathrm{B}+(\bar{c}-\kappa)\mathrm{C}\}+dt+\{a\mathrm{A}+\Delta b\,.\,\mathrm{B}+(\Delta c+\delta c)\,.\,\mathrm{C}\}$$

If now we take

$$\tau = t+\bar{b}\,.\,\mathrm{B}+(\bar{c}-\kappa)\mathrm{C}$$

where τ is the time of transit corrected in the ordinary way for level, collimation, and diurnal aberration, but without regard to pivot errors, we have

$$a = \tau+dt+a\,.\,\mathrm{A}+\{\Delta b\,.\,\mathrm{B}+(\Delta c+\delta c)\,.\,\mathrm{C}\}$$

Now Δb and Δc being constants, and δc as well as the factors B and C depending only on the position of the star, the last term in this may be tabulated with N.P.D. as argument. If we denote this quantity by Q, we have simply

$$a = \tau+dt+a\mathrm{A}+\mathrm{Q} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (3)$$

In general the azimuth constant is determined by comparing the observation of a star which culminates south of the zenith with that of a close polar star at upper or lower culmination. For the former we have

$$a = \tau+dt+a\,.\,\mathrm{A}+\mathrm{Q}$$

and similarly for the polar star

$$a' = \tau' + dt + a \,.\, \mathrm{A}' + \mathrm{Q}'$$

If, as is sometimes the case, the azimuth has been deduced from transits of two close polars, one above and the other below the pole, or from two transits of the same star at an interval of twelve hours, the first equation will apply to one, the second equation to the other transit. Thus in any case we have

$$a - a' = \tau - \tau' + a(\mathrm{A} - \mathrm{A}') + \mathrm{Q} - \mathrm{Q}'$$

or

$$a = \frac{(a - a') - (\tau - \tau')}{\mathrm{A} - \mathrm{A}'} - \frac{\mathrm{Q} - \mathrm{Q}'}{\mathrm{A} - \mathrm{A}'}$$

But $\frac{(a - a') - (\tau - \tau')}{\mathrm{A} - \mathrm{A}'}$ is the value of the azimuth constant determined in the usual way on the assumption of cylindrical pivots. If we denote it by $\bar{a}$ we have

$$a = \bar{a} + \Delta a$$

in which

$$\Delta a = -\frac{\mathrm{Q} - \mathrm{Q}'}{\mathrm{A} - \mathrm{A}'} \quad \ldots \quad \ldots \quad \ldots \quad (4)$$

Substituting $\bar{a} + \Delta a$ for a in equation (3) we have for any clock star

$$a_1 = \tau_1 + dt_1 + \bar{a}_1 \mathrm{A}_1 + \Delta a \,.\, \mathrm{A}_1 + \mathrm{Q}_1$$

Or if write $\overline{dt}_1 + \Delta t_1$ for dt_1, where $\overline{dt}_1$ is the clock correction deduced from this particular star without regard to pivot errors, while Δt_1 is a further correction required by the clock in consequence of these errors, we have

$$a_1 = \tau_1 + \overline{dt}_1 + \bar{a} \,.\, \mathrm{A}_1 + \Delta t_1 + \Delta a \,.\, \mathrm{A}_1 + \mathrm{Q}_1$$

But

$$\overline{dt}_1 = a_1 - \tau_1 - \bar{a} \,.\, \mathrm{A}_1$$

and therefore

$$\Delta t_1 = -\Delta a \,.\, \mathrm{A}_1 - \mathrm{Q}_1$$

Hence the mean value, Δt, of the further clock correction due to pivot errors, as deduced from all the clock stars employed on one night, is

$$\Delta t = -\Delta a \,.\, \frac{\Sigma \mathrm{A}}{n} - \frac{\Sigma \mathrm{Q}}{n}$$

where n is the number of stars employed.

Returning now to equation (3) we have for any star of the working list

$$\begin{aligned} a &= \tau + dt + a\mathrm{A} + \mathrm{Q} \\ &= (\tau + \overline{dt} + \bar{a} \,.\, \mathrm{A}) + \Delta t + \Delta a \,.\, \mathrm{A} + \mathrm{Q} \\ &= \bar{a} + \Delta a \left(\mathrm{A} - \frac{\Sigma \mathrm{A}}{n}\right) + \left(\mathrm{Q} - \frac{\Sigma \mathrm{Q}}{n}\right) \end{aligned}$$

where $\bar{a}$ is the value of the star's right ascension deduced without regard to pivot errors. If, further, we write $a = \bar{a} + \Delta a$, we have as the correction necessary to the already determined R.A.

$$\Delta a = \Delta a \,.\, \left(\mathrm{A} - \frac{\Sigma \mathrm{A}}{n}\right) + \left(\mathrm{Q} - \frac{\Sigma \mathrm{Q}}{n}\right) \quad \ldots \quad \ldots \quad (5)$$

We thus see that for new observations the correction for pivot errors may be taken from Table II., where it appears as a correction to the collimation constant depending upon the star's N.P.D., combined with the constant corrections Δb and Δc from equations (2) affecting the level and collimation constants respectively. When these corrections are applied the clock and azimuth errors deduced in the ordinary way will not be affected by the pivot errors. If, however, we seek the total correction to the already computed R.A. of a star it will be necessary to compute the quantities $\mathrm{Q} = \Delta b \,.\, \mathrm{B} + (\Delta c + \delta c) \,.\, \mathrm{C}$. With these quantities we must determine Δa by means of equation (4), and finally Δa by means of equation (5).

To be rigorously exact it would be necessary to compute Δa, and hence Δa, for every separate night of observation. Fortunately, however, it is found that the corrections so computed from night to night vary so slightly that a constant correction may with sensible accuracy be taken as applicable to any particular star.

From the values of δc given in Table II. we find by interpolation

$$\delta c_1 = +0''{\cdot}044 \,;\; \delta c_2 = -0''{\cdot}204 \,;\; \text{and } \delta c_3 = -0''{\cdot}072$$

Therefore

$$\Delta c = +0''{\cdot}014 \text{ and } \Delta b = -0''{\cdot}190.$$

We thus obtain the following table for Q :—

TABLE III.

Values of $\mathrm{Q} = \Delta b \,.\, \mathrm{B} + (\Delta c + \delta c) \,.\, \mathrm{C}$.

N.P.D.	Q	N.P.D.	Q	N.P.D.	Q	N.P.D.	Q
°	s	°	s	°	s	°	s
		−5	+0·181	+10	−0·047	+70	−0·054
−50	+0·008	4	·212	15	·023	75	·048
45	·017	3	·265	20	−0·002	80	·038
40	·026	2	·374	25	+0·054	85	·032
35	·036	−1	+0·719	30	·083	90	·029
30	·059	0		35	+0·031	95	·029
25	·076	+1	−0·716	40	−0·008	100	·019
20	·094	2	·367	45	·030	105	−0·001
15	·119	3	·254	50	·039	110	+0·010
−10	·123	4	·196	55	·049	115	·037
		+5	·156	60	·059	120	·056
				+65	·056	125	+0·030
						+130	−0·006

With these values the correction to the azimuth constant Δa was next computed from equation (4) for any pair of stars whose N.P.D.'s are found, one in the top row and the other in the left-hand column of Table IV. Thus, if the azimuth had been computed from a star of $-2°$ N.P.D. and another of $+90°$ N.P.D. the value of Δa would be $-0''{\cdot}34$. Seeing that all the polar stars which have been used at Oxford for azimuth are situated within less than 4° of the pole, and that the part of the correction Δa, which depends on Δa, does not amount to $0^s{\cdot}1$ even at 10° N.P.D., and is less than $0^s{\cdot}02$ from the zenith to the southern horizon, we see from the small differences in Table IV. that it will be sufficient to take a mean value of $-0''{\cdot}33$ as being the correction to the azimuth constant however determined.

TABLE IV.

Values of Δa.

N.P.D.	+1°.	+2°.	+3°.	+4°.	+5°.	+7°.	+9°.	+11°.
− 1°	−0″·30	−0″·31	−0″·31	−0″·31	−0″·32	−0″·33	−0″·32	−0″·30
− 2	−0·31	−0·31	−0·32	−0·32	−0·34	−0·36	−0·34	−0·31
− 3	−0·31	−0·32	−0·33	−0·33	−0·37	−0·40	−0·37	−0·33
− 4	−0·31	−0·34	−0·34	−0·35	−0·40	−0·45	−0·41	−0·35
+ 50	−0·29	−0·29	−0·29	−0·28				
+ 70	−0·28	−0·26	−0·25	−0·24				
+ 90	−0·29	−0·29	−0·29	−0·28				
+110	−0·30	−0·32	−0·33	−0·34				

Since the clock stars used both in the Radcliffe Catalogue for 1890 and in the observations made since the publication of that work are all contained between $+50°$ and $+110°$ N.P.D., it will be sufficient to take for $\frac{\Sigma Q}{n}$ the mean of the values of Q between those limits as given in Table III., and for $\frac{\Sigma A}{n}$ we may take the mean of the azimuth factors between the same limits. Thus we find

$$\frac{\Sigma Q}{n} = -0^s{\cdot}034 \quad \text{and} \quad \frac{\Sigma A}{n} = +0{\cdot}044$$

Substituting these values in equation (5) and putting $\Delta a = -0''{\cdot}33$ we find the values of Δa for every fifth degree of N.P.D. as given in Table V.

TABLE V.

Corrections for Irregularities of the Pivots to be applied to all Right Ascensions of Stars observed with the Transit Circle of the Radcliffe Observatory.

N.P.D.	Δα.	N.P.D.	Δα.	N.P.D.	Δα.	N.P.D.	Δα.
°	s	°	s	°	s	°	s
		−5	+0·057	+10	+0·062	+70	−0·018
−50	+0·028	4	·049	15	·060	75	·013
45	·034	3	·036	20	·067	80	−0·004
40	·041	2	+0·015	25	·115	85	+0·001
35	·048	−1	−0·030	30	·138	90	·002
30	·067	0		35	·082	95	·001
25	·078	+1	+0·095	40	·040	100	·010
20	·088	2	·054	45	·015	105	·027
15	·099	3	·037	50	+0·004	110	·036
−10	+0·076	4	·030	55	−0·008	115	·062
		+5	+0·031	60	·020	120	·079
				+65	−0·018	125	·052
						+130	+0·014

These are the corrections to be applied to all the right ascensions of the Radcliffe Catalogue for 1890 to free them from the effect of the irregularities of the pivots. Since the publication of that work a large number of right ascensions have been observed with the same instrument. These observations, which are now in preparation for the press, have been corrected by the application of the above quantities, and should be entirely free from the effect of pivot error.

As I have pointed out at the beginning of this paper, it was the existence of small but systematic differences between the right ascensions observed at Oxford and those observed with the Cape and Greenwich Transit Circles which in the first instance directed special attention to the form of the pivots of the Radcliffe instrument and gave rise to the present inquiry. It will be of interest therefore to compare these systematic differences with the corrections for pivot errors now deduced in a wholly independent manner. For this purpose I have given in Table VI. below the differences in R.A. multiplied by sin N.P.D., as found by Stone, between the Radcliffe Catalogue for 1890, and each of the two catalogues, Stone 1880, and Greenwich 1880, reprinted from his paper in the *Monthly Notices*, vol. lv. p. 295. In the last column I have added the corrections for pivot errors, also multiplied by sin N.P.D.

TABLE VI.

Group.	Mean N.P.D. of Group.	Differences in R.A. × sin N.P.D. Rad.—Stone.	Wt.	Rad.—Grn.	Wt.	Corrections for Pivot Errors.
° °	° ′	s		s		s
0 – 5				+0·014	636	+0·002
5 – 10				+0·024	17	+0·006
10 – 15				−0·017	54	+0·014
15 – 20				−0·014	71	+0·019
20 – 25				−0·017	64	+0·036
25 – 30				−0·002	62	+0·059
30 – 38¼				−0·005	55	+0·039
38¼ – 45	42 0	+0·015	4	+0·001	67	+0·020
45 – 50	46 0	+0·050	5	+0·027	44	+0·010
50 – 55	52 0	+0·045	11	+0·014	74	−0·001
55 – 60	57 30	+0·042	22	+0·031	69	−0·022
60 – 65	62 0	+0·056	118	+0·040	229	−0·017
65 – 70	68 0	+0·041	130	+0·036	296	−0·017
70 – 75	72 30	+0·057	151	+0·043	346	−0·015
75 – 80	77 30	+0·037	233	+0·038	405	−0·009
80 – 85	82 30	+0·034	293	+0·034	482	−0·002
85 – 90	87 30	+0·036	181	+0·028	352	+0·002
90 – 95	92 30	+0·017	397	+0·014	600	+0·002
95 –100	97 30	−0·004	352	−0·017	617	+0·006
100 –105	102 30	−0·021	259	−0·040	495	+0·018
105 –110	107 30	−0·024	277	−0·051	606	+0·030
110 –115	112 30	−0·053	642	−0·078	368	+0·045
115 –120	117 30	−0·064	303	−0·091	195	+0·062
120 –125½	122 30	−0·133	145	−0·158	45	+0·056

It will be seen from this table that in almost every case where the correction for pivot error amounts to as much as $0^s{\cdot}01$ it is of opposite sign to the corresponding difference of R.A. North of the zenith the correction is too big and more than cancels these differences, whilst on the other side of the zenith, and down to about 5° south of the equator, the correction is too small. From N.P.D. 95° down to 120° the corrections almost exactly account for the marked differences between the Radcliffe 1890 and Stone 1880, whilst the differences Radcliffe–Greenwich are largely reduced. The relatively large difference for the last group is, however, only very partially accounted for by pivot errors, and must be due to a combination of causes, one of which is obviously the very low altitude of these stars at Oxford and Greenwich.

Addendum.—Since the above was written I have received Professor Auwers' *Tafeln zur Reduction von Stern-Catalogen auf das System des Fundamentalcatalogs des Berliner Jahrbuchs.* From the comparison given in Table VII. it will be seen that the corrections depending on declination, $\Delta\alpha_\delta$, which he finds to be necessary to reduce the Oxford right ascensions to his system are, to a great extent, accounted for by the pivot errors as determined in this paper.

Table VII.

N.P.D.	Red. to Fund. Cat.	Corr. for Pivot Error.	N.P.D.	Red. to Fund. Cat.	Corr. for Pivot Error.
°	°	s	°	s	s
10	+0·125	+0·062	70	−0·003	−0·018
15	+0·113	+0·060	75	0·000	−0·013
20	+0·095	+0·067	80	+0·005	−0·004
25	+0·075	+0·115	85	+0·014	+0·001
30	+0·056	+0·138	90	+0·026	+0·002
35	+0·040	+0·082	95	+0·041	+0·001
40	+0·027	+0·040	100	+0·062	+0·010
45	+0·018	+0·015	105	+0·084	+0·027
50	+0·009	+0·004	110	+0·101	+0·036
55	+0·002	−0·008	115	+0·120	+0·062
60	−0·001	−0·020	120	+0·137	+0·079
65	−0·003	−0·018			

The Positions of Seventy Stars in the Cluster M 13 Herculis.

By H. C. Plummer, M.A.

The complete investigation, so far as it can be made by photographic means, of a dense cluster, such as *M* 13 *Herculis*, must naturally be based on plates taken with a telescope of great focal length and high resolving power. To secure such plates is in fact one of the important functions of the very largest refractors. But the large-scale photographs thus obtained may cover so small a region of the sky that they contain no stars whose meridian places are known and by which the reduction of the plates can therefore be made. In this case it is necessary to determine the positions of a sufficient number of reference stars by some auxiliary means. This has been done for the cluster mentioned at the request of the director of the Liverpool Observatory, who is at present discussing a photograph taken with the Yerkes refractor.

The Oxford plate (No. 2372) was taken with the astrographic instrument on 1904 September 17. An exposure of eighty

minutes was given (from $19^h\ 50^m$ to $21^h\ 10^m$ sidereal time). The adopted plate centre is

$$1900{\cdot}0 \begin{cases} \text{R.A.} & 16^h\ 38^m\ 0^s \\ \text{Decl.} & +36^\circ\ 40'\ 0'' \end{cases}$$

The comparison stars (thirty-three in number) are taken from the *Lund A.G. Catalogue*, and the relations between their computed coordinates (ξ, η) and the corresponding measured coordinates (x, y) are found to be

$$x-\xi = -0{\cdot}00065\ x + 0{\cdot}00395\ y - 0{\cdot}1628$$
$$y-\eta = -0{\cdot}00421\ x - 0{\cdot}00050\ y + 0{\cdot}4014$$

Table I. gives the results of the comparison with the Lund meridian places. The first column gives the A.G.C. number and the second and third columns give the standard coordinates for 1900·0, calculated from the catalogue with the unit $5' = 1$ réseau interval and increased by 13. The fourth and fifth columns contain the residuals in R.A. and Declination in the sense of *corrections* to the Lund places.

The stars whose positions are given in Table II. extend over a field which is the same as that covered by Plate 34 in the late Dr. Roberts' *Stars, Star Clusters, and Nebulæ*, vol. i., and have all been identified on this plate. The reference number is in order of R.A. The columns headed x and y contain the standard coordinates for 1900·0, expressed in minutes of arc and referred to the centre above stated. The fourth and fifth columns give the deduced R.A. and Declination for the epoch 1900·0. The measures of the cluster stars have been made in four orientations of the plate by means of one of the ordinary Oxford micrometers, in which an eye-piece scale gives direct readings to 0·001 of a $5'$ réseau interval. A certain want of definition in the images of these stars made it appear better to multiply the number of independent settings on the star images than to increase the mere accuracy of the readings.

TABLE I.

Lund A.G.C.	ξ.	η.	$\Delta\alpha$. s	$\Delta\delta$. ″
6817	0·7127	7·9992	+0·06	−0·3
6818	1·8326	18·4231	+0·22	+0·4
6822	3·3115	25·3513	+0·01	−0·5
6823	3·7436	23·1591	−0·16	−0·5
6825	4·9867	21·1180	+0·07	+2·2
6828	5·8256	21·8189	−0·04	−0·2
6829	5·7759	6·1078	+0·01	+0·2
6830	5·9472	3·2107	−0·06	+0·2
6833	7·4238	1·7660	+0·01	−0·5

Lund A.G.C.	ξ.	η.	$\Delta\alpha$. s	$\Delta\delta$. ″
6836	8·2174	13·2583	+0·05	−1·1
6838	8·4315	1·5274	+0·03	+0·9
6840	9·2697	25·5471	−0·09	0·0
6844	9·9932	23·6048	−0·10	0·0
6846	10·2429	19·7579	+0·08	+0·2
6848	10·5654	3·4090	−0·14	+0·1
6850	11·7567	9·7306	+0·02	−1·1
6851	12·0784	19·0228	+0·02	−0·3
6857	14·5183	8·3966	+0·11	+0·3
6863	16·6194	13·3717	+0·02	−0·7
6864	17·1310	19·8546	+0·08	+0·1
6865	18·0451	11·8143	−0·04	−0·5
6867	19·9075	23·7595	+0·12	−0·1
6868	20·1439	10·3230	−0·02	−0·6
6869	20·7401	4·0765	−0·07	+1·2
6870	20·8000	4·6149	+0·03	−0·4
6876	22·8420	20·2366	+0·01	−1·7
6878	23·1917	2·7869	+0·08	+1·6
6880	23·3958	16·2414	−0·10	+1·2
6882	24·2654	15·5270	+0·03	−0·8
6883	24·1711	23·4301	−0·20	−1·0
6884	24·7864	6·9385	−0·03	−0·6
6885	24·9923	16·3677	−0·15	−0·3
6886	24·9972	19·1082	+0·12	+2·5

TABLE II.

Ref. No.	1900·0. x.	y.	1900·0. R.A. h m s	Decl. ° ′ ″
1	− 6′·958	−0′·408	16 37 25·30	+36 39 35·2
2	− 5·696	−3·098	37 31·62	36 53·9
3	− 5·620	+5·046	37 31·94	45 2·6
4	− 5·582	−0·249	37 32·16	39 44·9
5	− 5·258	+1·605	37 33·77	41 36·1
6	− 4·805	−6·881	37 36·08	33 7·0
7	− 3·920	+4·878	37 40·43	44 52·6
8	− 3·874	−5·217	37 40·70	34 46·9
9	− 3·811	+0·907	37 40·99	40 54·3
10	− 3·594	−8·882	16 37 42·11	36 31 7·0

Ref. No.	1900·0. x.	1900·0. y.	1900·0. R.A. h m s	1900·0. Decl. ° ′ ″
	′	′		
11	− 3·474	−2·694	16 37 42·69	+36 37 18·3
12	− 3·031	−7·248	37 44·91	32 45·1
13	− 2·974	−1·907	37 45·18	38 5·5
14	− 2·944	−3·865	37 45·33	36 8·0
15	− 2·926	+2·591	37 45·40	42 35·4
16	− 2·652	−0·534	37 46·78	39 27·9
17	− 2·117	+1·760	37 49·44	41 45·6
18	− 2·073	−3·000	37 49·67	37 0·0
19	− 1·957	−2·753	37 50·25	37 14·8
20	− 1·663	−9·208	37 51·72	30 47·5
21	− 1·652	−5·678	37 51·77	34 19·3
22	− 1·436	−0·520	37 52·84	39 28·8
23	− 1·346	+5·422	37 53·28	45 25·3
24	− 1·266	−3·060	37 53·69	36 56·4
25	− 0·866	+3·400	37 55·68	43 24·0
26	− 0·732	+1·191	37 56·35	41 11·5
27	− 0·664	−2·547	37 56·69	37 27·2
28	− 0·442	−0·035	37 57·80	39 57·9
29	− 0·398	−0·950	37 58·02	39 3·0
30	− 0·376	−3·827	37 58·13	36 10·4
31	− 0·127	−2·411	37 59·37	37 35·3
32	+ 0·332	−0·727	38 1·66	39 16·4
33	+ 0·377	−3·372	38 1·88	36 37·7
34	+ 0·472	+2·316	38 2·36	42 19·0
35	+ 0·813	−0·037	38 4·06	39 57·8
36	+ 0·834	−5·871	38 4·15	34 7·7
37	+ 0·870	−2·910	38 4·34	37 5·4
38	+ 1·392	+0·264	38 6·94	40 15·8
39	+ 1·389	+5·644	38 6·94	45 38·6
40	+ 1·664	−2·082	38 8·30	37 55·1
41	+ 1·811	−2·104	38 9·03	37 53·8
42	+ 1·904	−5·997	38 9·48	34 0·2
43	+ 2·266	+2·559	38 11·31	42 33·5
44	+ 2·278	+1·629	38 11·36	41 37·7
45	+ 2·284	+1·416	16 38 11·39	36 41 24·9

Ref. No.	1900·0. x.	y.	1900·0. R.A.	Decl.
	′	′	h m s	° ′ ″
46	+ 2·550	−2·607	16 38 12·71	+36 37 23·5
47	+ 2·654	−0·090	38 13·24	39 54·5
48	+ 3·186	−4·306	38 15·87	35 41·6
49	+ 3·196	−4·881	38 15·92	35 7·1
50	+ 3·228	+1·140	38 16·10	41 8·3
51	+ 3·352	+4·739	38 16·73	44 44·3
52	+ 3·360	+0·332	38 16·76	40 19·9
53	+ 3·572	−2·096	38 17·80	37 54·2
54	+ 3·576	−5·911	38 17·81	34 5·3
55	+ 4·092	−1·206	38 20·40	38 47·5
56	+ 4·228	+0·438	38 21·08	40 26·2
57	+ 4·315	+4·864	38 21·54	44 51·7
58	+ 4·504	+1·461	38 22·47	41 27·5
59	+ 4·620	−3·228	38 23·02	36 46·2
60	+ 4·636	+5·332	38 23·14	45 19·8
61	+ 5·045	−0·819	38 25·15	39 10·7
62	+ 5·308	−5·020	38 26·44	34 58·7
63	+ 5·624	+3·228	38 28·06	43 13·5
64	+ 6·722	+1·002	38 33·53	40 59·8
65	+ 7·110	−6·172	38 35·41	33 49·4
66	+ 8·166	−1·300	38 40·71	38 41·6
67	+ 8·222	−0·669	38 41·00	39 19·4
68	+ 8·468	−3·642	38 42·20	36 21·0
69	+ 8·900	+0·666	38 44·39	40 39·4
70	+10·003	−0·938	16 38 49·88	36 39 3·1

University Observatory, Oxford:
1904 *November* 8.

Note on the Variation of ε Aurigæ. By Colonel E. E. Markwick.

Dr. Ludendorff, of Potsdam, has recently published a paper on the variability of ε *Aurigæ*, in which he reaches some rather remarkable conclusions. Having discussed a considerable number of observations of the brightness of this star made by observers of repute, commencing with those of Argelander in 1842, he

finds that the light-variation is of the Algol-type. The extraordinary part of the result is the great length of the period—$54\frac{1}{4}$ years. Minima occur, or follow one another, after 9905^{d}, or 27·12 years. As they are possibly unequal in brightness, it is concluded they represent the passing of the lesser star between us and the primary, followed by a passing of the same behind or the other side of the primary. The duration of the light change is 1·99 years, divided symmetrically as follows:—

	d.
From cessation of normal light to commencement of minimum	207
Duration of minimum	313
From end of minimum to resumption of normal light	207
	727

Total period of light change = 1·99 years.

The middle epoch of last minimum occurred in 1902 March 31.

The variation, as given by Ludendorff, amounts to $0^{m}\cdot73$, or from $3^{m}\cdot35$ to $4^{m}\cdot08$ (see *Astron. Nach.* 3918–19–20).

Taking the *Provisional Catalogue of Variable Stars*, published by Harvard College Observatory, we find that the periods of the Algol-type stars range from $31^{d}\cdot3$, the longest, to $0^{d}\cdot5$, the shortest. Even the former is exceptionally long—far more so than the average. The $54\frac{1}{4}$-year period is therefore most remarkable by comparison.

As ε *Aurigæ* has been on my working list for a number of years, I thought it would be of interest to inquire whether my work bears out the results just alluded to. My observations number in all 126, extending from 1888 March 18 to 1904 May 2. In addition I have received from Mr. W. E. Besley four observations made in the years 1895 and 1896. These 130 determinations of brightness were made visually by Argelander's method, and are reduced to the scale of the *Revised Harvard Photometry*. They are given at the end of this note.

In the first two observations the comparison star used was θ *Aurigæ*. From 1888 November 30 to 1897 January 30 η was generally the comparison star, with occasionally θ in addition. From 1897 March 1 to 1899 η, ζ, and ι. From 1899 October 23 onwards η and ι only, except in 1902.

The observations from the commencement to 1892 March 8 were made in Ireland; from 1895 January 29 to 1898 May 10 in Gibraltar; thereafter in England. Those marked * are referred to by Ludendorff in his paper.

Certain notes from the original observing books are entered in the column headed Remarks. It was not thought necessary to encumber the list with the detailed light-comparisons.

The magnitudes of the comparison stars adopted are as follows, being taken from the R.H.P.

	m.
η Aurigæ	3·26
θ „	2·70
ζ „	3·80
ι „	2·99

I may say that the whole of the observations were made practically with the naked eye.

Before proceeding further I should like to remark that little or no importance is attached to the second decimal in the deduced magnitudes of *ε Aurigæ*. The comparisons were generally made in tenths of a magnitude; the two decimals come out when deducing the brightness from the magnitudes of the comparison stars which are given to two places of decimals in the R.H.P.

The observations, as contained in the list, have been plotted on square-ruled paper, the ordinate representing one magnitude in brightness corresponding in length to the abscissa for one-third of a year. Three methods have been adopted: 1. The magnitudes were all plotted direct from the list, and the points joined. The result is a series of zigzag lines for each season, all of which, with the exception of three observations in the year 1902, run fairly near the line representing $3^m{\cdot}0$. The three observations of 1902, however, are much below this, their mean being practically $4^m{\cdot}0$. For all the period under discussion, excluding 1902, the magnitudes are absolutely comprised within the limits 2·47 and 3·53. It is noticed incidentally that observations in April or May are usually higher (brighter) than any others. These curves are not reproduced here.

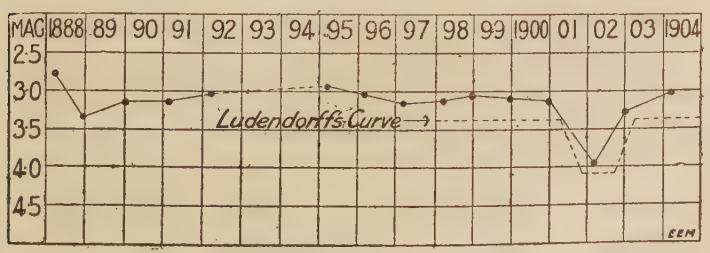

Fig. 1.

2. In order to smooth the asperities in the curves or lines just referred to, the mean brightness for each "season" has been taken, and the results plotted. This is represented in fig. 1. A glance at this diagram shows the brightness of the star to have been fairly constant, except in the season 1901–02, when there was a marked diminution of light corresponding to about 1 m.

Expressed in figures, the mean brightness for seasons is as follows :—

Season.	Magnitude.	No. of Observations.	Season.	Magnitude.	No. of Observations.
1888	2·80	2	1897	3·17	11
1888–1889	3·35	11	1898	3·11	15
1889–1890	3·18	8	1898–1899	3·08	19
1891	3·12	7	1899–1900	3·10	15
1892	3·05	8	1900–1901	3·13	12
Interval			1902	3·97	3
1895	2·96	1	1902–1903	3·26	5
1896	3·06	4	1904	3·04	9

Superposed in fig. 1 is the light curve as deduced by Ludendorff, and it is at once seen that my observations support his result exactly, so far as the year 1902 is concerned. I regret that for some reason or another I did not make more observations in 1902 and 1903, whence the shape of the light curve might have been more accurately determined for the period of the light fluctuation. I have no observations bearing on the previous minimum in 1874–1875.

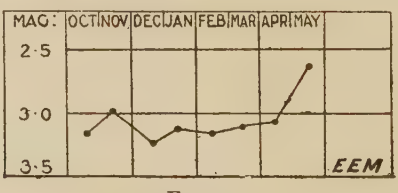

FIG. 2.

I may add that the mean of four observations made thi season (1904) is $3^{m}\cdot13$, or about "normal" light.

3. Reference is made above to the apparent increased brightness of ε *Aurigæ* in May. I have therefore taken out the mean brightness by months, with the exception of the year 1902. The following is the result :—

Month.	Mean brightness.	No. of Observations.	Month.	Mean brightness.	No. of Observations.
October	3·18	4	February	3·19	21
November	3·00	4	March	3·11	40
December	3·26	15	April	3·09	16
January	3·14	24	May	2·64	3

These magnitudes are plotted in fig. 2. It is seen that from the beginning of the year until the end of the season there is an apparent tendency for the brightness to increase.

This increase is, I believe, subjective, and due to the difficult conditions under which the star is observed in April and May.

ε Aurigæ is a circumpolar star for the latitude of London, and as the year advances it can only be seen low down on the northern horizon with a background of bright twilight. One would imagine that with a brighter background there would be a tendency to make or estimate the star fainter than usual on this very account. Our curve, however, indicates the reverse of this; and as the observations are differential, or made by comparison with other stars, I am rather inclined to attribute the apparent increase in brightness to "position angle"—that is, to the different angle made with the vertical by the line joining the variable and the comparison star compared with the same angle when the constellation is higher in the sky. For example, I have noted more than once when observing in May that *ι Aurigæ* is getting vertically below ε, and immersed in the mists of the horizon.

There is a moral in all this for variable-star observers. Do not tire in watching a variable such as the one now in question, or *α Cassiopeiæ*, *α Orionis*, &c. One may observe for years without any change, and when one least expects it an important and marked change may occur. Although, according to Ludendorff's result, no further change is due for twenty-five years, yet I would urge observers to keep a watch on *ε Aurigæ* with a view to confirming the remarkable result already announced.

Observations of (Ch. 1768) ε Aurigæ.

Date.	Mag.	Remarks.
1888.		
Mar. 18	2·8	
30	2·8	
Nov. 30	3·0	
Dec. 8	3·53	
24	3·53	
29	3·4	
1889.		
Jan. 29	3·4	
Feb. 20	3·46	
27	3·26	
Mar. 4	3·4	
5	3·4	
Apr. 17	3·4	
21	3·06	
Dec. 18	3·4	
20	3·1	
24	3·1	

Date.	Mag.		Remarks.
1890.			
Jan. 23	3·16	*	
Feb. 13	3·1	*	
20	3·16	*	
Mar. 19	3·16	*	
29	3·3	*	
1891.			
Jan. 9	3·4	*	
Mar. 2	3·06	*	
5	3·16	*	
8	2·93	*	
10	3·08	*	
20	3·1	*	Moonlight
Apr. 2	3·08	*	
1892.			
Jan. 1	3·0	*	
3	3·1	*	
5	3·0	*	Moonlight

Date.	Mag.	Remarks.
1892.		
Jan. 24	3·06 *	
27	3·01 *	
29	3·1 *	
Feb. 24	3·1 *	
Mar. 8	3·0 *	Moonlight
Interval		
1895.		
Jan. 29	2·96	
1896.		
Jan. 15	3·16	
Feb. 20	3·06	
Mar. 3	29·6	Sky hazy
17	3·06	
1897.		
Jan. 1	3·01	
30	3·26	
Mar. 1	3·17	
5	3·17	
9	3·17	Moonlight
20	3·07	Moonlight
21	3·17	
24	3·22	
25	3·22	
Apr. 5	3·33	Moonlight
8	3·07	Moonlight
1898.		
Jan. 16	3·27	
21	3·17	Position awkward, near zenith
23	3·22	
25	3·27	
Feb. 15	3·40	
24	3·17	Moonlight
25	3·22	Moonlight
Mar. 13	3·22	
23	3·08	
25	2·98	Moonlight
Apr. 15	2·98	ε and η at nearly same altitude
21	2·92	ε and η at nearly same altitude
22	2·95	
23	2·95	Moonlight
May 10	2·78	
Dec. 19	3·30	Moonlight
20	3·32	Moonlight
22	3·12	Moonlight
29	2·95	
1899.		
Jan. 14	2·72	
26	3·12	Moonlight
Feb. 2	3·15	Moonlight
22	2·92	
27	3·12	Sky poor
28	3·07	
Mar. 1	3·10	
2	2·88	Sky poor
4	3·08	
5	3·22	
9	3·22	
13	2·98	
14	3·12	
15	3·05	Moonlight
Apr. 3	3·12	
Oct. 23	3·22	Moonlight
28	3·18	
31	3·15	
Nov. 10	2·82	Moonlight
1900.		
Jan. 25	3·27	
Feb. 7	3·17	Moonlight
Mar. 4	3·27	
17	3·07	Moonlight
30	2·97	Very poor sky
31	3·30	
Apr. 17	3·22	
21	3·27	Very bad sky
26	2·92	Twilight
28	3·02	Twilight
May 7	2·67	Moonlight
Oct. 21	3·17	
Nov. 18	3·12	

Date.	Mag.	Remarks.
1900. Nov. 27	3·06	
Dec. 13	3·18	Observations by W. E. Besley
15	3·18	
19	3·18	
21	3·17	
1901. Feb. 5	3·02	Moonlight
14	3·02	
15	3·07	
Mar. 15	3·07	
22	3·27	ι seemed very bright
1902. Apr. 16	4·0±	"Reliable, I think; but it is a colder whiter tint than ζ"
May 4	3·70	Bright twilight
10	4·20	Compared only with ζ, as ι was too near the horizon
Dec. 24	3·39	
1903. Jan. 23	3·17	
28	3·20	
Feb. 23	3·31	
Mar. 3	3·25	"Most difficult to judge"
1904. Jan. 16	3·22	
19	3·22	
Feb. 6	3·12	Mean of two observations
13	3·17	
15	3·02	
Mar. 10	3·02	
Apr. 9	2·92	
12	3·22	ι looked reddish
May 2	2·47	Bright twilight

Telescopic Observation of a Meteor Trail. By W. Shackleton, A.R.C.Sc. (Lond.)

Whilst making a search for Encke's Comet towards midnight of October 12 I was startled by the field of view becoming suddenly illuminated, and on moving the telescope a few minutes in right ascension a bright ribbon-like streak came at once into view, which I was astonished to find sinuous and double. Glancing momentarily to the sky a long trail, evidently left by a meteor, was visible; moreover the trail appeared perfectly straight as far as the naked eye could judge. There was no doubt, however, that the trail, when observed under a magnifying power of 46, was irregular, the deviations bearing a strong resemblance to the path of an electric spark through air, except that the contortions were smaller and less rigid than is usually shown when the electric current takes the path of least resistance. The field of view of the telescope was 40′·5, and on tracing the trail by moving the telescope some three or four kinks were visible in each succeeding field.

The path of the meteor lay nearly in a circle of right ascension, and a movement of the telescope in declination showed that the whole trail was marked by these undulations, with here

and there knotty globular patches, as if the meteor in its flight had shed occasional larger masses of incandescent matter.

The trail was tubular or consisted of two parallel narrow ribbons, each component being about twice the angular diameter of *Jupiter*, or about 90″ in width, separated by an interval of about 5″. The trail remained feebly visible to the naked eye for some minutes, but telescopic observation showed it to be diffusing into nebulous patches, with sufficient luminosity to render comet-seeking futile from the time of first observation, 11.39 P.M., until midnight.

The position of the telescope at the time of appearance was R.A. $0^h\ 45^m$, Dec. $+28°$. Approximate position of trail was from Dec. $+15°$ to $+35°$, in a circle of R.A. at $0^h\ 45^m$.

Ephemeris for Physical Observations of the Moon for 1905.
By A. C. D. Crommelin.

Greenwich Midnight. 1905.	Selenographical Colong. of the Sun.	Selenographical Lat. of the Sun.	Geocentric Libration. Sel. Long. of the Earth.	Geocentric Libration. Lat. of the Earth.	Physical Libration. Long.	Physical Libration. Lat.	C.
	°	°	°	°	°	°	°
Jan. 1	219·25	+1·32	+3·84	−6·29	+·004	+·004	14·34
2	231·43	+1·31	+4·42	−6·58	+·004	+·004	9·11
3	243·61	+1·30	+4·83	−6·49	+·003	+·005	3·32
4	255·80	+1·28	+5·01	−6·05	+·003		357·40
5	267·98	+1·27	+4·94	−5·29	+·002		351·72
6	280·17	+1·26	+4·57	−4·28	+·001		346·61
7	292·37	+1·24	+3·91	−3·08	·000		342·31
8	304·55	+1·23	+2·99	−1·75	−·002		338·97
9	316·73	+1·21	+1·83	−0·37	−·004		336·65
10	328·91	+1·20	+0·51	+1·02	−·006		335·37
11	341·08	+1·19	−0·89	+2·35	−·008		335·11
12	353·25	+1·17	−2·29	+3·58	−·010		335·86
13	5·41	+1·15	−3·60	+4·67	−·012		337·59
14	17·54	+1·14	−4·72	+5·57	−·014		340·31
15	29·71	+1·12	−5·57	+6·23	−·016	+·005	343·97
16	41·86	+1·10	−6·09	+6·61	−·018	+·006	348·53
17	53·99	+1·08	−6·22	+6·67	−·019		353·86
18	66·13	+1·06	−5·94	+6·37	−·019		359·74
19	78·25	+1·03	−5·29	+5·69	−·019		5·85
20	90·38	+1·01	−4·31	+4·65	−·020		11·73
21	102·50	+0·98	−3·08	+3·30	−·019		16·91
22	114·63	+0·96	−1·71	+1·72	−·018	+·006	20·99

Greenwich Midnight.	Selenographical Colong. of the Sun.	Lat. of the Sun.	Geocentric Libration. Sel. Long.	Lat. of the Earth.	Physical Libration. Long.	Lat.	C.
1905.	°	°	°	°	°	°	°
Jan. 23	126·76	+0·93	−0·30	+0·02	−·018	+·006	23·69
24	138·90	+0·91	+1·06	−1·68	−·017		24·86
25	151·04	+0·88	+2·30	−3·25	−·016		24·51
26	163·18	+0·85	+3·38	−4·61	−·015	+·006	22·70
27	175·34	+0·83	+4·26	−5·67	−·015	+·007	19·61
28	187·50	+0·80	+4·94	−6·37	−·015		15·41
29	199·67	+0·78	+5·39	−6·71	−·014		10·37
30	211·85	+0·76	+5·61	−6·67	−·014		4·78
31	224·03	+0·73	+5·61	−6·28	−·014		358·98
Feb. 1	236·22	+0·71	+5·37	−5·57	−·015		353·30
2	248·41	+0·69	+4·91	−4·60	−·015		348·07
3	260·60	+0·66	+4·21	−3·42	−·017		343·54
4	272·80	+0·64	+3·31	−2·09	−·018		339·91
5	284·99	+0·62	+2·22	−0·69	−·020		337·28
6	297·18	+0·60	+0·97	+0·73	−·022		335·67
7	309·37	+0·57	−0·39	+2·10	−·024	+·007	335·09
8	321·56	+0·55	−1·79	+3·38	−·026	+·008	335·54
9	333·74	+0·53	−3·18	+4·52	−·028		336·97
10	345·92	+0·50	−4·49	+5·47	−·030		339·35
11	358·10	+0·48	−5·63	+6·20	−·032		342·64
12	10·26	+0·45	−6·52	+6·66	−·033		346·81
13	22·42	+0·42	−7·10	+6·81	−·034		351·75
14	34·58	+0·40	−7·30	+6·63	−·035		357·32
15	46·73	+0·37	−7·08	+6·10	−·035		3·24
16	58·87	+0·34	−6·43	+5·19	−·035		9·19
17	71·01	+0·31	−5·36	+3·95	−·035		14·71
18	83·15	+0·27	−3·95	+2·41	−·035		19·35
19	95·28	+0·24	−2·28	+0·69	−·034	+·008	22·72
20	107·41	+0·21	−0·48	−1·10	−·033	+·009	24·58
21	119·55	+0·18	+1·32	−2·81	−·032		24·79
22	131·69	+0·15	+3·00	−4·31	−·031		23·41
23	143·84	+0·12	+4·45	−5·50	−·030		20·59
24	156·00	+0·09	+5·58	−6·33	−·029		16·55
25	168·16	+0·06	+6·37	−6·75	−·028		11·59
26	180·33	+0·03	+6·80	−6·78	−·028		6·06
27	192·51	0·00	+6·87	−6·45	−·028		0·26
28	204·69	−0·02	+6·61	−5·80	−·029		354·55
Mar. 1	216·89	−0·05	+6·07	−4·87	−·029	+·009	349·25

Greenwich Midnight.	Selenographical Colong. of the Sun.	Lat.	Geocentric Libration. Sel. Long. of the Earth.	Lat.	Physical Libration. Long.	Lat.	C
1905.	°	°	°	°	°	°	°
Mar. 2	229·08	−0·08	+5·29	−3·73	−·030	+·009	344·60
3	241·28	−0·10	+4·30	−2·44	−·031		340·77
4	253·49	−0·13	+3·15	−1·05	−·032		337·88
5	265·69	−0·15	+1·89	+0·37	−·034		336·0
6	277·90	−0·18	+0·54	+1·77	−·036	+·009	335·16
7	290·11	−0·20	−0·85	+3·09	−·038	+·010	335·33
8	302·31	−0·22	−2·24	+4·27	−·040		336·4
9	314·51	−0·25	−3·59	+5·27	−·041		338·61
10	326·71	−0·27	−4·84	+6·05	−·042		341·65
11	338·91	−0·30	−5·95	+6·57	−·043		345·53
12	351·10	−0·33	−6·85	+6·80	−·045		350·16
13	3·28	−0·36	−7·49	+6·73	−·045		355·42
14	15·46	−0·38	−7·80	+6·32	−·045		1·09
15	27·63	−0·41	−7·73	+5·56	−·045		6·88
16	39·80	−0·44	−7·24	+4·48	−·044		12·47
17	51·96	−0·47	−6·32	+3·09	−·044		17·41
18	64·11	−0·50	−4·99	+1·46	−·043		21·34
19	76·26	−0·53	−3·30	−0·29	−·042		23·91
20	88·41	−0·56	−1·36	−2·06	−·041		24·88
21	100·56	−0·59	+0·69	−3·69	−·039		24·18
22	112·71	−0·62	+2·69	−5·05	−·038	+·010	21·88
23	124·87	−0·65	+4·48	−6·04	−·037	+·011	18·16
24	137·03	−0·67	+5·93	−6·61	−·036		13·32
25	149·19	−0·70	+6·95	−6·75	−·036		7·74
26	161·37	−0·72	+7·50	−6·50	−·035		1·82
27	173·55	−0·75	+7·60	−5·91	−·035		355·94
28	185·75	−0·77	+7·27	−5·03	−·035		350·45
29	197·94	−0·79	+6·60	−3·93	−·035		345·61
30	210·14	−0·81	+5·64	−2·67	−·036		341·58
31	222·36	−0·83	+4·47	−1·32	−·037		338·48
Apr. 1	234·57	−0·85	+3·17	+0·08	−·038		336·37
2	246·78	−0·87	+1·78	+1·47	−·039		335·28
3	259·01	−0·89	+0·38	+2·78	−·040		335·21
4	271·23	−0·91	−1·01	+3·98	−·042		336·12
5	283·45	−0·93	−2·35	+5·01	−·043		338·02
6	295·67	−0·95	−3·59	+5·82	−·045	+·011	340·84
7	307·89	−0·97	−4·72	+6·39	−·046	+·012	344·53
8	20·11	−0·99	−5·71	+6·65	−·047	+·012	348·97

Greenwich Midnight.	Selenographical Colong. of the Sun.	Lat.	Geocentric Libration. Sel. Long. of the Earth.	Lat.	Physical Libration. Long.	Lat.	C.
1905.	°	°	°	°	°	″	°
Apr. 9	332·33	−1·01	−6·53	+6·65	−·048	+·012	354·04
10	344·53	−1·03	−7·13	+6·32	−·048		359·54
11	356·74	−1·05	−7·47	+5·67	−·048		5·11
12	8·94	−1·07	−7·51	+4·70	−·048		10·73
13	21·13	−1·09	−7·20	+3·46	−·047		15·77
14	33·31	−1·12	−6·50	+1·98	−·045		19·97
15	45·49	−1·14	−5·40	+0·33	−·044		23·02
16	57·66	−1·16	−3·92	−1·38	−·043		24·65
17	69·83	−1·18	−2·13	−3·02	−·041		24·71
18	81·99	−1·20	−0·12	−4·47	−·040		23·12
19	94·16	−1·22	+1·92	−5·60	−·039	+·012	19·99
20	106·32	−1·24	+3·84	−6·33	−·037	+·013	15·51
21	118·49	−1·26	+5·46	−6·62	−·035		10·05
22	130·67	−1·27	+6·65	−6·47	−·035		4·02
23	142·85	−1·29	+7·35	−5·95	−·034		357·89
24	155·04	−1·30	+7·55	−5·12	−·034		352·07
25	167·23	−1·31	+7·27	−4·05	−·033		346·89
26	179·44	−1·32	+6·59	−2·81	−·034		342·55
27	191·64	−1·33	+5·59	−1·47	−·034		339·18
28	203·86	−1·34	+4·37	−0·09	−·035		336·81
29	216·08	−1·35	+3·01	+1·27	−·035		335·47
30	228·31	−1·36	+1·59	+2·58	−·037		335·14
May 1	240·54	−1·37	+0·18	+3·77	−·038	+·013	335·81
2	252·77	−1·38	−1·15	+4·80	−·038	+·014	337·47
3	265·01	−1·39	−2·38	5·63	−·040		340·07
4	277·24	−1·40	−3·48	6·22	−·041		343·57
5	289·48	−1·41	−4·42	6·53	−·041		347·87
6	301·72	−1·42	−5·20	6·54	−·042		352·84
7	313·95	−1·43	−5·81	6·24	−·042		358·28
8	326·18	−1·44	−6·24	5·63	−·041		3·93
9	338·41	−1·45	−6·47	4·72	−·041		9·47
10	350·63	−1·46	−6·47	3·55	−·040		14·57
11	2·84	−1·47	−6·22	2·15	−·039		18·92
12	15·04	−1·48	−5·66	+0·61	−·038		22·22
13	27·25	−1·49	−4·79	−1·01	−·036		24·26
14	39·44	−1·49	−3·58	−2·60	−·034		24·86
15	51·63	−1·50	−2·08	−4·04	−·033	+·014	23·94
16	63·81	−1·51	−0·36	−5·23	−·031	+·015	21·48

Greenwich Midnight.	Selenographical Colong. of the Sun.	Lat. of the Sun.	Geocentric Libration. Sel. Long. of the Earth.	Lat. of the Earth.	Physical Libration. Long.	Lat.	C.
1905.	°	°	°	°	°	°	°
May 17	75·99	−1·52	+1·45	−6·06	−·029	+·015	17·60
18	88·17	−1·52	+3·20	−6·48	−·028		12·53
19	100·35	−1·52	+4·73	−6·46	−·027		6·63
20	112·53	−1·53	+5·90	−6·03	−·026		0·38
21	124·72	−1·53	+6·63	−5·25	−·025		354·25
22	136·91	−1·53	+6·86	−4·20	−·024		348·65
23	149·11	−1·53	+6·63	−2·96	−·024		343·89
24	161·31	−1·53	+6·00	−1·61	−·023		240·11
25	173·52	−1·52	+5·03	−0·22	−·024	+·015	337·40
26	185·74	−1·52	+3·82	+1·15	−·025	+·016	335·75
27	197·97	−1·52	+2·48	+2·46	−·026		335·14
28	210·20	−1·51	+1·08	+3·66	−·027		335·54
29	222·43	−1·51	−0·28	+4·70	−·028		336·93
30	234·67	−1·51	−1·53	+5·54	−·029		339·28
31	246·91	−1·50	−2·64	+6·15	−·029		342·53
June 1	259·16	−1·50	−3·55	+6·48	−·030		346·65
2	271·41	−1·49	−4·27	+6·52	−·030		351·50
3	283·66	−1·49	−4·79	+6·24	−·031		356·89
4	295·91	−1·49	−5·11	+5·65	−·030		2·58
5	308·15	−1·49	−5·25	+4·75	−·029		8·24
6	320·39	−1·49	−5·22	+3·59	−·028		13·52
7	332·63	−1·48	−5·02	+2·21	−·027		18·05
8	344·86	−1·48	−4·63	+0·69	−·025		21·58
9	357·09	−1·48	−4·05	−0·90	−·024		23·89
10	9·30	−1·47	−3·25	−2·45	−·022		24·84
11	21·51	−1·47	−2·23	−3·87	−·020		24·35
12	33·72	−1·46	−1·02	−5·07	−·018		22·42
13	45·92	−1·45	+0·34	−5·95	−·016		19·10
14	58·11	−1·45	+1·77	−6·45	−·014		14·54
15	70·30	−1·44	+3·14	−6·53	−·012		9·02
16	82·50	−1·43	+4·34	−6·19	−·011	+·016	2·91
17	94·68	−1·42	+5·25	−5·48	−·010	+·017	356·68
18	106·87	−1·40	+5·80	−4·47	−·009		350·77
19	119·06	−1·39	+5·93	−3·24	−·009		345·59
20	131·26	−1·38	+5·66	−1·86	−·009		341·35
21	143·46	−1·36	+5·02	−0·44	−·008		338·32
22	155·67	−1·35	+4·07	+0·98	−·009		336·18
23	167·88	−1·33	+2·90	+2·33	−·009	+·017	335·23

Greenwich Midnight.	Selenographical Colong. of the Sun.	Lat. of the Sun.	Geocentric Libration. Sel. Long.	Lat. of the Earth.	Physical Libration. Long.	Lat.	C.
1905.	°	°	°	°	°	°	°
June 24	180·11	−1·32	+1·60	+3·57	−·010	+·017	335·32
25	192·33	−1·30	+0·25	+4·64	−·011		336·42
26	204·56	−1·28	−1·06	+5·52	−·011		338·47
27	216·80	−1·27	−2·24	+6·16	−·012		341·45
28	229·04	−1·25	−3·25	+6·54	−·013		345·29
29	241·29	−1·24	−4·02	+6·63	−·013		349·92
30	253·54	−1·23	−4·54	+6·39	−·013		355·19
July 1	265·79	−1·21	−4·79	+5·84	−·013		0·87
2	278·05	−1·20	−4·80	+4·96	−·012		6·65
3	290·30	−1·18	−4·59	+3·79	−·011		12·16
4	302·55	−1·17	−4·20	+2·40	−·010		17·01
5	314·80	−1·16	−3·64	+0·85	−·009		20·87
6	327·04	−1·14	−2·94	−0·77	−·007		23·49
7	339·27	−1·13	−2·13	−2·36	−·005		24·75
8	351·50	−1·11	−1·23	−3·81	−·003	+·017	24·57
9	3·72	−1·10	−0·23	−5·04	−·001	+·018	22·98
10	15·94	−1·08	+0·82	−5·96	+·001		20·05
11	28·15	−1·06	+1·89	−6·51	+·003		15·90
12	40·35	−1·04	+2·92	−6·66	+·005		10·77
13	52·55	−1·02	+3·84	−6·41	+·006		4·94
14	64·74	−1·00	+4·58	−5·78	+·008		358·82
15	76·93	−0·97	+5·08	−4·83	+·009		352·84
16	89·13	−0·95	+5·29	−3·62	+·009		347·38
17	101·32	−0·93	+5·17	−2·25	+·010		342·78
18	113·51	−0·90	+4·74	−0·79	+·010		339·22
19	125·71	−0·88	+4·01	+0·68	+·010		336·77
20	137·91	−0·85	+3·02	+2·08	+·009		335·45
21	150·12	−0·83	+1·86	+3·38	+·009		335·20
22	162·33	−0·80	+0·57	+4·49	+·008		336·02
23	174·54	−0·78	−0·75	+5·46	+·007		337·73
24	186·76	−0·75	−2·03	+6·16	+·007		340·40
25	198·99	−0·73	−3·18	+6·61	+·006		343·93
26	211·23	−0·70	−4·14	+6·77	+·006	+·018	348·26
27	223·46	−0·68	−4·84	+6·62	+·005	+·019	353·28
28	235·71	−0·66	−5·25	+6·14	+·005		358·82
29	247·96	−0·64	−5·34	+5·34	+·005		4·61
30	260·21	−0·62	−5·11	+4·22	+·006		10·29
31	272·45	−0·59	−4·58	+2·84	+·007	+·019	15·47

Greenwich Midnight.	Selenographical Colong. of the Sun.	Lat.	Geocentric Libration. Sel. Long. of the Earth.	Lat.	Physical Libration. Long.	Lat.	O.
1905.	°	°	°	°	°	°	°
Aug. 1	284·70	−0·57	−3·79	+1·27	+·008	+·019	19·77
2	296·95	−0·55	−2·80	−0·41	+·010		22·86
3	309·20	−0·53	−1·68	−2·08	+·012		24·54
4	321·44	−0·51	−0·48	−3·62	+·013		24·73
5	333·67	−0·49	+0·72	−4·93	+·015		23·44
6	345·90	−0·46	+1·88	−5·93	+·017		20·77
7	358·12	−0·44	+2·93	−6·55	+·019		16·89
8	10·33	−0·41	+3·85	−6·77	+·021		12·00
9	22·54	−0·39	+4·59	−6·59	+·022		6·40
10	34·74	−0·36	+5·12	−6·04	+·023		0·44
11	46 93	−0·33	+5·42	−5·16	+·024		354·50
12	59·12	−0·30	+5·47	−4·01	+·025		348·96
13	71·31	−0·27	+5·28	−2·67	+·025		344·13
14	83·50	−0·24	+4·83	−1·22	+·026		340·26
15	95·68	−0·21	+4·15	+0·25	+·025	+·019	337·46
16	107·87	−0·18	+3·25	+1·71	+·025	+·020	335·77
17	120·06	−0·15	+2·18	+3·06	+·024		335·18
18	132·25	−0·12	+0·97	+4·26	+·023		335·65
19	144·45	−0·09	−0·32	+5·27	+·022		337·09
20	156·65	−0·06	−1·64	+6·05	+·021		339·47
21	168·85	−0·03	−2·91	+6·57	+·020		342·70
22	181·06	−0·01	−4·07	+6·82	+·019		346·74
23	193·28	+0 02	−5·05	+6·76	+·018		351·47
24	205·50	+0·05	−5·78	+6·39	+·018		356·75
25	217·73	+0·07	−6·19	+5·71	+·018		2·38
26	229·96	+0·10	−6·25	+4·71	+·018		8·08
27	242·19	+0·12	−5·92	+3·42	+·019		13·47
28	254·43	+0·14	−5·21	+1·90	+·020		18·17
29	266·67	+0·16	−4·14	+0·22	+·021		21·80
30	278·91	+0·19	−2·78	−1·51	+·022		24·07
31	291·15	+0·21	−1·22	−3·15	+·023		24·82
Sept. 1	303·39	+0·24	+0·43	−4·59	+ 025		23·98
2	315·62	+0·26	+2·04	−5·71	+·027		21·64
3	327·84	+0·28	+3·51	−6·45	+·028	+·020	17·96
4	340·06	+0·31	+4·74	−6·77	+·030	+·021	13·21
5	352·27	+0·33	+5·68	−6·67	+0·32		7·69
6	4·48	+0·36	+6·29	−6·18	+·033		1·76
7	16·67	+0·39	+6·57	− 5·36	+ ·034	+·021	355·82

Greenwich Midnight.	Selenographical Colong.	Lat. of the Sun.	Geocentric Libration. Sel. Long.	Lat. of the Earth.	Physical Libration. Long.	Lat.	C.
1905.	°	°	°	°	°	°	°
Sept. 8	28·87	+0·42	+6·54	−4·28	+·035	+·021	350·22
9	41·05	+0·45	+6·23	−2·99	+·035		345·27
10	53·23	+0·48	+5·67	−1·58	+·035		341·19
11	65·41	+0·51	+4·91	−0·12	+·035		338·13
12	77·58	+0·54	+3·97	+1·33	+·036		336·15
13	89·76	+0·57	+2·89	+2·70	+·035		335·26
14	101·93	+0·60	+1·70	+3·94	+·033		335·42
15	114·11	+0·62	+0·43	+4·99	+·032		336·59
16	126·29	+0·65	−0·88	+5·83	+·031		338·71
17	138·46	+0·68	−2·19	+6·41	+·030		341·70
18	150·64	+0·70	−3·46	+6·72	+·029		345·48
19	162·84	+0·73	−4·63	+6·74	+·028	+·021	349·96
20	175·03	+0·75	−5·64	+6·47	+·027	+·022	355·00
21	187·23	+0·77	−6·44	+5·89	+·026		0·43
22	199·43	+0·79	−6·95	+5·02	+·026		6·00
23	211·64	+0·82	−7·10	+3·87	+·026		11·41
24	223·85	+0·84	−6·85	+2·47	+·027		16·33
25	236·07	+0·85	−6·15	+0·89	+·027		20·39
26	248·30	+0·87	−5·01	−0·80	+·028		23·24
27	260·52	+0·89	−3·46	−2·48	+·030		24·67
28	272·75	+0·91	−1·62	−4·01	+·031		24·50
29	284·97	+0·93	+0·39	−5·27	+·032		22·71
30	297·19	+0·95	+2·40	−6·16	+·034		19·42
Oct. 1	309·41	+0·97	+4·22	−6·62	+·035		14·86
2	321·63	+0·99	+5·74	−6·62	+·036		9·36
3	333·83	+1·01	+6·84	−6·21	+·037		3·35
4	346·03	+1·03	+7·49	−5·45	+·038		357·25
5	358·22	+1·05	+7·70	−4·40	+·039		351·46
6	10·41	+1·07	+7·51	−3·16	+·039		346·32
7	22·59	+1·10	+6·98	−1·78	+·039		342·04
8	34·76	+1·12	+6·19	−0·35	+·039		338·76
9	46·93	+1·14	+5·19	+1·07	+·038		336·54
10	59·09	+1·16	+4·05	+2·43	+·037		335·39
11	71·25	+1·18	+2·82	+3·66	+·036		335·30
12	83·41	+1·20	+1·53	+4·73	+·035		336·21
13	95·57	+1·22	+0·23	+5·59	+·033		338·09
14	107·73	+1·24	−1·07	+6·21	+·031		340·87
15	119·89	+1·26	−2·34	+6·56	+·030	+·022	344·45

Greenwich Midnight. 1905.	Selenographical Colong. of the Sun.	Lat.	Geocentric Libration. Sel. Long. of the Earth.	Lat.	Physical Libration. Long.	Lat.	C.
	°	°	°	°	°	°	°
Oct. 16	132·05	+1·27	−3·56	+6·62	+·028	+·023	348·75
17	144·21	+1·29	−4·70	+6·40	+·027		353·63
18	156·38	+1·30	−5·70	+5·89	+·027		358·91
19	168·55	+1·32	−6·53	+5·10	+·026		4·37
20	180·73	+1·33	−7·12	+4·05	+·025		9·73
21	192·91	+1·34	−7·40	+2·77	+·025		14·70
22	205·10	+1·35	−7·30	+1·30	+·026		18·97
23	217·29	+1·36	−6·77	−0·28	+·026		22·23
24	229·49	+1·37	−5·76	−1·89	+·027		24·22
25	241·70	+1·38	−4·30	−3·43	+·028		24·76
26	253·90	+1·39	−2·45	−4·77	+·029		23·71
27	266·12	+1·40	−0·33	−5·79	+·030		21·08
28	278·33	+1·40	+1·86	−6·39	+·031		16·98
29	290·53	+1·41	+3·92	−6·53	+·033		11·70
30	302·74	+1·42	+5·68	−6·22	+·033		5·65
31	314·94	+1·43	+6·99	−5·52	+·034		359·32
Nov. 1	327·13	+1·44	+7·79	−4·50	+·034		353·19
2	339·32	+1·45	+8·08	−3·26	+·034		347·67
3	351·50	+1·47	+7·89	−1·89	+·034		343·05
4	3·67	+1·48	+7·32	−0·46	+·033		339·47
5	15·84	+1·49	+6·43	+0·95	+·032		336·98
6	28·00	+1·49	+5·33	+2·30	+·032		335·58
7	40·16	+1·50	+4·08	+3·53	+·030		335·29
8	52·31	+1·51	+2·77	+4·59	+·028		335·92
9	64·45	+1·51	+1·44	+5·45	+·027		337·56
10	76·60	+1·52	+0·14	+6·08	+·026		340·11
11	88·74	+1·52	−1·11	+6·45	+·024		343·51
12	100·88	+1·53	−2·29	+6·53	+·022	+·023	347·66
13	113·03	+1·53	−3·39	+6·33	+·021	+·024	352·43
14	125·17	+1·53	−4·40	+5·84	+·019		357·63
15	137·31	+1·53	−5·29	+5·07	+·018		3·09
16	149·46	+1·53	−6·03	+4·06	+·017		8·46
17	161·62	+1·53	−6·58	+2·83	+·017		13·49
18	173·78	+1·53	−6·89	+1·43	+·016		17·87
19	185·94	+1·52	−6·89	−0·08	+·017		21·35
20	198·11	+1·52	−6·52	−1·62	+·017		23·70
21	210·29	+1·51	−5·74	−3·11	+·018		24·72
22	222·47	+1·51	−4·53	−4·44	+·018	+·024	24·32

Greenwich Midnight.	Selenographical Colong.	Lat. of the Sun.	Geocentric Libration. Sel. Long.	Lat. of the Earth.	Physical Libration. Long.	Lat.	C.
1905.	°	°	°	°	°	°	°
Nov. 23	234·66	+1·50	−2·93	−5·51	+·019	+·024	22·51
24	246·84	+1·50	−1·02	−6·22	+·020		19·00
25	259·05	+1·50	+1·03	−6·50	+·021		14·26
26	271·24	+1·49	+3·04	−6·32	+·021		8·48
27	283·44	+1·49	+4·82	−5·71	+·022		2·11
28	295·63	+1·48	+6·22	−4·73	+·022		355·67
29	307·82	+1·48	+7·13	−3·49	+·023		349·69
30	320·01	+1·48	+7·54	−2·09	+·022		344·55
Dec. 1	332·18	+1·48	+7·45	−0·62	+·022		340·50
2	344·36	+1·47	+6·95	+0·84	+·021		337·60
3	356·52	+1·47	+6·10	+2·23	+·020		335·87
4	8·69	+1·46	+5·01	+3·48	+·018		335·25
5	20·84	+1·45	+3·76	+4·57	+·017		335·66
6	32·98	+1·44	+2·43	+5·45	+·015	+·024	337·06
7	45·13	+1·44	+1·11	+6·09	+·013	+·025	339·36
8	57·27	+1·43	−0·16	+6·47	+·011		342·54
9	69·40	+1·42	−1·34	+6·58	+·009		346·50
10	81·54	+1·40	−2·40	+6·39	+·007		351·14
11	93·67	+1·39	−3·34	+5·91	+·005		356·30
12	105·80	+1·38	−4·13	+5·15	+·005		1·76
13	117·93	+1·36	−4·78	+4·13	+·004		7·23
14	130·06	+1·34	−5·28	+2·90	+·003		12·41
15	142·20	+1·33	−5·62	+1·49	+·003		16·98
16	154·34	+1·31	−5·76	−0·01	+·002		20·66
17	166·48	+1·29	−5·76	−1·54	+·003		23·24
18	178·64	+1·28	−5·30	−3·01	+·003		24·58
19	190·80	+1·26	−4·64	−4·34	+·003		24·57
20	202·96	+1·24	−3·66	−5·43	+·004		23·17
21	215·13	+1·22	−2·38	−6·20	+·005		20·38
22	227·31	+1·20	−0·87	−6·57	+·006		16·27
23	239·50	+1·19	+0·78	−6·52	+·006		11·02
24	251·68	+1·17	+2·42	−6·02	+·008		4·96
25	263·87	+1·16	+3·91	−5·14	+·008	+·025	358·52
26	276·06	+1·14	+5·13	−3·93	+·008	+·026	352·25
27	288·25	+1·12	+5·96	−2·52	+·008		346·62
28	300·44	+1·11	+6·36	−0·99	+·007		342·00
29	312·62	+1·09	+6·34	+0·54	+·006		338·56
30	324·80	+1·07	+5·91	+2·01	+·005		336·36
31	336·97	+1·05	+5·15	+3·35	+·004	+·026	335·36

The longitudes are reckoned in the plane of the Moon's equator, the axis of reference being the radius which passes through the mean centre of the visible disc. This axis therefore rotates with the Moon, and is not fixed in space.

The inclination of the Moon's equator to the ecliptic is taken as 1°·523, the value used in the *Nautical Almanac*.

The physical librations in longitude and latitude, as given by Professor Franz's formulæ, have been applied; their values are also printed separately, so that those who prefer to use Hayn's coefficients (*Ast. Nach.* 3956) can do so. His longitude coefficient is about one quarter of Franz's. Thus to reduce to Hayn's value we apply three-quarters of the printed physical libration in longitude with its own sign to Sun's colongitude, and with reversed sign to selenographical longitude of the Earth.

The colongitude of the Sun is 90° (or 450°) *minus* his selenographical longitude. It is numerically equal to the selenographical longitude of the morning terminator reckoned eastward from the mean centre of the disc. Hence its value is approximately 270°, 0°, 90°, 180° at new Moon, first quarter, full Moon, last quarter respectively. The longitude of the evening terminator is of course 180° greater or less than that of the morning one.

When the geocentric libration in longitude is positive, the region brought into view is on the west limb; when negative, on the east.

When the geocentric libration in latitude is positive, the region brought into view is at the Moon's north pole; when negative, at the south.

As it did not appear that anyone made use of the "Combined Amount" columns they have been omitted. They can, of course, be derived from the librations in Longitude and Latitude by a method analogous to the transformation from rectangular to polar coordinates.

C denotes the geocentric position-angle of the northern extremity of the Moon's axis measured eastward from the northernmost point of the disc. It has been computed by the second formula given in the Preface to the *Nautical Almanac*.

The terms "East" and "West" are used throughout with reference to our sky, and not as they would appear to an observer on the Moon.

I give the method for finding the altitude of the Sun at a given point on the Moon whose position is defined: (1) by selenographical longitude and latitude; (2) by direction cosines.

In either case the Sun's selenographical colongitude and latitude (K, L supposed) must be found by interpolation from the ephemeris for the given time.

In the first case let the given point be in the position longitude M, latitude N. Longitudes are reckoned from the meridian passing through the mean centre of the disc, and the

positive direction is that towards the Mare Crisium. North latitudes are considered positive.

Then

$$\text{sine Sun's altitude} = \sin \mathrm{L} \sin \mathrm{N} + \cos \mathrm{L} \cos \mathrm{N} \sin (\mathrm{K}+\mathrm{M}).$$

In the second case let ξ, η, ζ be the direction cosines of the given point. The axes are (1) that diameter of the Moon's equator which is 90° from the mean centre of the disc ; (2) the Moon's polar axis ; (3) the diameter through the mean centre of the disc. The positive directions are as above. Mr. Saunder has issued some maps of portions of the Moon's surface from which the co-ordinates ξ, η, ζ can be taken at sight.

Then the Sun's direction cosines are :

$$\cos \mathrm{K} \cos \mathrm{L}, \sin \mathrm{L}, \sin \mathrm{K} \cos \mathrm{L},$$

and sine Sun's altitude

$$= \xi \cos \mathrm{K} \cos \mathrm{L} + \eta \sin \mathrm{L} + \zeta \sin \mathrm{K} \cos \mathrm{L}.$$

Neither formula is convenient when the Sun's altitude is very great, for an angle near 90° cannot be accurately determined from its sine. However, when the Sun is high the shadows are so inconspicuous that it is not necessary to compute his altitude with great accuracy.

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